Naval Research Laboratory

Washington, DC 20375-5000

NRL Memorandum Report 6148

Radar Information from the Partial Derivatives of the Echo Signal Phase from a Point Scatterer

MERRILL I. SKOLNIK

Radar Division

February 17, 1988

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY 2b. DECLASSIFICATION / DOWNGRADING SCHEDULE	3. DISTRIBUTION / AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
4. PERFORMING ORGANIZATION REPORT NUMBER(S) NRL Memorandum Report 6148	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION Naval Research Laboratory 6b. OFFICE SYMBOL (If applicable) Code 5300	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code) Washington, DC 20375-5000	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING / SPONSORING ORGANIZATION Office of the Chief of Naval Research 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code) 800 North Quincy Street Arlington, VA 22217-5000	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO.: 61153N PROJECT NO.: TASK NO: WORK UNIT ACCESSION NO.:

11. TITLE (Include Security Classification)
Radar Information from the Partial Derivatives of the Echo Signal Phase from a Point Scatterer

12. PERSONAL AUTHOR(S)
Skolnik, Merrill I.

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
	FROM ______ TO ______	1988 February 17	55

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

This report is concerned with the target information from a point scatterer as obtained form the partial derivatives of the echo-signal phase as a function of frequency, time, and spatial position. These partial derivates provide the three basic radar measurements of range, range rate, and angle. The angle rate found from the spatial doppler-frequency shift (due to a scatterer with a component of velocity perpendicular to the radar line of sight) is suggested as a fourth basic radar measurement. The direct measurement of angle-rate from the spatial doppler frequency is illustrated by the example of a scanning interferometer radar with a cosine radiation pattern. The relationship between the partial phase-derivative measurements and the more conventional radar measurements is shown.

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21 ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Merrill I. Skolnik	22b TELEPHONE (Include Area Code) (202) 767-2936 22c OFFICE SYMBOL 5300

DD Form 1473, JUN 86 Previous editions are obsolete

S/N 0102-LF-014-6603

Contents

Summary

This report describes the measurement of range, range rate, angle, and angle rate of a single point-scatterer as obtained from the partial derivatives of the received echo-signal phase as a function of frequency f, time t, and position x, as follows:

$$\left.\frac{\partial\phi}{\partial f}\right|_{x,t} \longrightarrow \text{range} \qquad \left.\frac{\partial\phi_s}{\partial x}\right|_{f,t} \longrightarrow \text{angle}$$

$$\left.\frac{\partial\phi}{\partial t}\right|_{f,x} \longrightarrow \text{range rate} \qquad \left.\frac{\partial\phi_s}{\partial t}\right|_{f,x} \longrightarrow \text{angle rate}$$

where ϕ is a temporal phase (Eq 3 of the text) equal to $2\pi fT$, T is the time delay to the target and back, and ϕ_s is a spatial phase (Eq. 7) equal to $2\pi(x/\lambda)\sin\xi$, x is the characteristic antenna dimension (in a simple interferometer it is the distance between the two antennas), λ is the radar wavelength, and ξ is the angle of arrival. The angle rate is included along with the more usual measurements of range, range rate, and angle since it can be obtained from a spatial doppler-frequency shift, analogous to the use of the temporal doppler-frequency shift that provides the range rate.

It is shown (Eq. 11) that the spatial doppler-frequency shift obtained by an interferometer antenna of characteristic dimension x is $f_s = (v_c/\lambda)(x/R)$, where v_c is the cross velocity (or tangential velocity) and R is the range. Although the spatial doppler-frequency shift is derived assuming an interferometer antenna generating a spatial cosine-wave pattern (grating lobes), it is shown that the spatial doppler-frequency shift will cause an expansion or contraction of a conventional single-beam antenna radiation pattern just as the temporal doppler shift results in an expansion or contraction of a time waveform. If the angle-rate and the range R are known, the cross velocity v_c can be determined since the angle rate is v_c/R. A knowledge of both the radial velocity (from the temporal doppler) and the cross velocity (from the spatial doppler) permits the vector velocity (speed and direction) of a point scatterer to be determined.

The scanning radar interferometer is used in this report to illustrate the extraction of the spatial doppler-frequency shift. In some cases, the spatial doppler can be obtained on a single scan of the interferometer pattern past the target. In most practical cases, however, a longer observation time is necessary. The spatial doppler may be obtained from observations on successive antenna scans, similar to how a conventional MTI radar recognizes the temporal doppler-frequency shift produced by a radially moving target from observations made on successive sweeps. Blind speeds can occur with spatial MTI processing, but they differ from the temporal blind speeds of the classical MTI radar.

The use of matched-filter detection of spatial doppler-frequency-shifted signals is examined in both the temporal domain and the antenna aperture domain. It was found that the spatial doppler-frequency shift usually

obtained with a scanning interferometer radar observing aircraft targets requires only a single filter and not a bank of matched filters for detection of signals with unknown spatial doppler. A bank of spatial matched filters might still be needed, however, when the total time of observation is large, as when a moving target traverses the pattern of a fixed (non-scanning) antenna.

Radar measurements based on partial phase derivatives are different from measurements made by conventional radar. The connection between the two is made in this report. Starting with the two-frequency CW measurement of range to a single scatterer ($\Delta\phi/\Delta f$), which is either highly ambiguous or very inaccurate, it is shown qualitatively how additional frequency components can produce an accurate and unambiguous range measurement of a single point scatterer. The further addition of frequency components to provide a "filled" spectrum allows an accurate, unambiguous measurement of range to multiple scatterers. It is also shown that the range measurement of N scatterers can be had by measuring $\Delta\phi/\Delta f$ for N independent pairs of frequency.

Expressions for the rms accuracy of the range-rate and angle-rate measurements are given. They show that measurement of range rate based on temporal doppler frequency can be much more accurate than the measurement of the rate of change of range based on two measurements spaced a finite time apart. Similarly, the measurement of angle-rate based on the spatial doppler frequency can be more accurate than the measurement of the rate of change of angle with time. It is found that the angle-rate accuracy depends on the extent of the antenna radiation pattern in space.

This report has shown the connection between classical radar measurements from a point scatterer and the measurements derived from the partial derivatives of phase from the echo signal. This has led to a better appreciation of the spatial character of radar signals; in particular, the spatial doppler frequency due to cross velocity and the measurement of angle rate based on the spatial doppler.

Acknowledgment

This report was typed by Mrs. Margaret Hornsby of the Division Administrative Staff. Her patience and professional excellence in this task are greatly appreciated by the author.

RADAR INFORMATION FROM THE PARTIAL DERIVATIVES OF THE ECHO SIGNAL PHASE FROM A POINT SCATTERER

1. BASIC RADAR INFORMATION

The traditional role of radar has been to detect the presence of targets, to determine their location in range and angle, and to infer something about their nature. Initially, radar was essentially a "blob" detector. Targets were detected and located, but little else could be said about them since one blob looked like any other blob when viewed on a conventional PPI radar display. If the target were tracked so that its speed and trajectory were found, something could be learned about its nature; i.e., that it was a ship and not a high-speed aircraft. Over the years, improvements in signal processing, waveform design, and a better understanding of radar signals have allowed the extraction of more information about the target. For example, high range resolution, using either a short pulse or a pulse compression waveform, provided the radial "profile" of a distributed target. The doppler frequency shift, which is extensively used in radar for detecting moving targets in the presence of clutter, was applied to give cross-range resolution when there was relative motion between radar and target (as in synthetic aperture radar). Modulation of the echo cross section due to mechanical motions of the target (such as propeller or jet engine modulation) gave information about the character of the target. Radar measurements made with orthogonal polarizations have also been explored for target recognition.

Table 1 summarizes the information that has been considered in the past to be potentially available from radar. The information is grouped according to (1) the target considered as a "point" scatterer, (2) the target considered as a distributed scatterer, and (3) the surface properties of a target. (A "point" scatterer is, of course, a fiction; but it is a convenient concept when the effects of a finite-size scatterer need not be considered.) The listing in this table represents what has been achieved or what is thought likely, but one finding of the present study is that this list is not complete.

In addition to adding to our knowledge of radar capability, understanding of the basic target information available from a radar signal is of practical interest for (1) the remote sensing of the environment and (2) noncooperative target recognition (NCTR). (A noncooperative target is one which is not in direct communication with the radar; it is recognized by actions taken unilaterally by the radar.) Remote sensing is generally, but not always, a civilian application. NCTR is generally, but not always, a military application.

Radar has been employed or considered as a remote sensor of the environment for many applications, as shown by the partial listing in Table 2. Weather radar is a good example of the increasing effectiveness of remote sensing. The early weather radars, such as the National Weather Service's WSR-57, can only measure the geometrical extent and strength of precipitation. New meteorological radars such as Nexrad, that extract the doppler frequency and its spectral spread, are able to provide considerably more weather information. They permit forecasting the appearance of severe meteorological conditions such as mesocyclones, tornadoes, hail, flooding, and downbursts that are a threat to aircraft.

As a recognizer of noncooperative targets, radar can distinguish one type of target from another (e.g., a ship from an airplane) or recognize different classes of the same type of target (e.g., a Starling from a Mallard). There are several levels of NCTR, as listed in Table 3.

Manuscript approved October 19, 1987.

Table 1 - Information from a Radar Target

- o <u>Target as a Point Scatterer</u>
  - range
  - range rate (doppler frequency shift)
  - angle
- o <u>Target as a Distributed Scatterer</u>
  - size
  - shape
  - change of shape
  - symmetry (polarization response)
- o <u>Surface Properties of a Target</u>
  - roughness
  - dielectric constant

Table 2 - Radar Applications of Remote Sensing

Atmosphere

- weather
- ionosphere
- ornithology
- entomology

Land

- geological prospecting
- agriculture (soil moisture, crop census)
- mapping (land use)

Sea

- geoid measurement
- sea state, sea spectrum, surface wind
- ice mapping

Underground

- detection of utility pipes and cables
- subsurface anomalies

Extraterrestrial

- meteors, aurora
- planetary exploration

Table 3 - The Several Levels of Noncooperative Target Recognition with Radar

Kind of target

- ship, aircraft, land clutter, motor vehicle, cloud, chaff, bird, etc.

General nature of target

- military or civil

Type of target

- fighter vs bomber
- cargo ship vs tanker
- armored vehicle vs truck
- chaff vs ship

Class of target

- F-14 vs MIG-23
- DD 963 vs FFG-7
- Starling vs Mallard (birds)

Identification

- name or side number (not usually possible with radar)

Thus there are important civilian and military applications which require the extraction of the information from radar echo signals. Understanding the basic information available from a radar signal allows radar to be used to its full potential.

The purpose of the study of Radar Information, of which this report is a part, is to provide a general treatment of the information potentially available from radar, to describe the basic nature of radar measurements, and to identify the factors that affect them. The initial approach is based on the amplitude and phase spectra of the target-scattered (echo) signal. The variation of the amplitude and phase of the echo as a function of frequency, time, and spatial position provides information about the target. In addition to including current radar measurements and the information they provide, as outlined in Table 1, two measurements not usually employed with radar have been added. These are (1) the angle rate, or cross velocity, and (2) the change (with time) of the profile in the cross-range dimension.

The present report deals chiefly with the information that can be obtained when the target is considered as a point scatterer. The information available from a point scatterer is its location and the rate of change of location. It will be shown that location and the rate of change of location are given by the partial derivatives of the scattered signal phase as a function of frequency, spatial position, and time. The measurement of angle-rate is included, but the change of cross-range profile and other measurements characteristic of a distributed target are not treated here.

2. AMPLITUDE AND PHASE SPECTRA

The fact that the amplitude and phase spectra of the radar echo can provide a basis for radar information extraction was first advanced in a short paper by R. J. Lees, then with the Royal Radar Establishment in England. (His paper apparently is not well known. It is briefly summarized in Appendix I.) Lees's approach is intere[illegible] in that it allows radar measurements and the extraction of information [illegible] viewed from a different perspective than normally considered by radar engineers. He proposed that the phase derivatives of the echo signal are associated with the measurements of range, range rate, and angle. As was indicated in Table 1, these three measurements assume that a target is a "point" scatterer. The amplitude variation* as a function of frequency, spatial position, and time was said by Lees to provide the measurements of the size, shape, and change of shape of a target considered as a distributed scatterer. The description of target information provided by Lees (Table 4) is not complete and not fully accurate, but he was probably the first to identify the role of the amplitude and phase spectra in the basic understanding of radar measurements. The correct interpretations of the amplitude variations are given in the parentheses in Table 4.

Consider the transmitted signal to have an amplitude a_t and a carrier frequency f_0. The phase is taken to be zero. The transmitted signal is a

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* (Lees used the term "amplitude derivative"; instead of "amplitude variation," which is not quite correct.)

Table 4

Information from Phase and Amplitude Spectra

Phase Derivatives*	Amplitude Variations**
$\left.\frac{\partial\Phi}{\partial f}\right\vert_{t,x} \longrightarrow$ range	$A(f) \longrightarrow$ size (radial profile)
$\left.\frac{\partial\Phi}{\partial t}\right\vert_{f,x} \longrightarrow$ range rate	$A(t) \longrightarrow$ change of shape (change of radial profile)
$\left.\frac{\partial\Phi}{\partial x}\right\vert_{f,t} \longrightarrow$ angle	$A(x) \longrightarrow$ shape (cross-range, or angle, profile)

* The expression $\partial\Phi/\partial f|_{t,x}$ means the partial derivative of phase Φ with respect to frequency f, at a fixed time t, and spatial position x.

** As originally stated by Lees. The correct interpretations are given in parentheses.

single pulse of sine wave of frequency f, represented as

$$s_t(t) = a_t \sin 2\pi ft \qquad (1)$$

(The pulse is assumed to be of finite duration and a constant amplitude, but the pulse width does not directly enter in this analysis.) After scattering by the target, the signal received back at the radar will be modified in both amplitude and phase by the nature of the target and its distance from the radar. The received signal can be described as having an amplitude spectrum and a phase spectrum. (For a single point scatterer the amplitude spectrum is the same as that which was transmitted.)

The emphasis in this report is on the information available from a single point scatterer. When the signal of Eq. 1 is transmitted, the received signal from a point scatterer is

$$s_r(t) = a_r \sin [2\pi f(t-T)] = a_r \sin (2\pi ft-\phi) \qquad (2)$$

where $\phi = 2\pi fT$, and $T = 2R/C$ = transit time to target at range R and back, c = velocity of propagation, and the received amplitude is a_r. It is the partial derivatives of the phase ϕ that will provide the target location information and the rate of change of location. Only the first derivatives of phase are considered here. (In Appendix II it is shown that the imaginary part of the received signal spectrum from a point scatterer is the same as the ϕ in Eq 2.)

3. PHASE DERIVATIVES

Frequency derivative - The partial derivative of phase as a function of frequency, at a particular time t and spatial position x, is a measure of the range (distance) to the target. Consider, as in Fig. 1, a radar illuminating a target at a range R. If the transmitted signal is of the form $a_t \sin 2\pi ft$, and if the target can be considered as a point scatterer, the received signal is that given by Eq. 2. We need only consider the phase of the argument of Eq. 2, which is $\phi = 2\pi fT$. The rate of change of phase with frequency $\partial\phi/\partial f = 2\pi T$; thus, the variation of phase with frequency is the measurement of time delay, or range. (Range = $cT/2$, where c = velocity of propagation.)

In principle, a measurement of the phase $\phi = 2\pi fT$ at a single frequency can provide the range directly, without the need to take the partial derivative. However, such a measurement is highly ambiguous, since phase cannot be unambiguously determined if it is greater than 2π radians. Thus the product fT must be less than unity for an unambiguous phase measurement. Since $T = 2R/c$, the maximum unambiguous range is one-half wavelength with a single measurement of phase. This is an impractical limitation at microwave frequencies.

Time derivative. The phase $\phi = 2\pi fT$ is a function of time since T (and the range R) varies when the target is in motion. When ϕ is written as $4\pi R/\lambda$, then $\partial\phi/\partial t = (4\pi/\lambda)\ \partial R/\partial t$. Hence, the derivative of phase with respect to time gives the radial velocity. As indicated in Fig. 1, the range is expressed as $R = R_0 - v(\cos\theta)t$, where R_0 is the range at $t = 0$, v is the

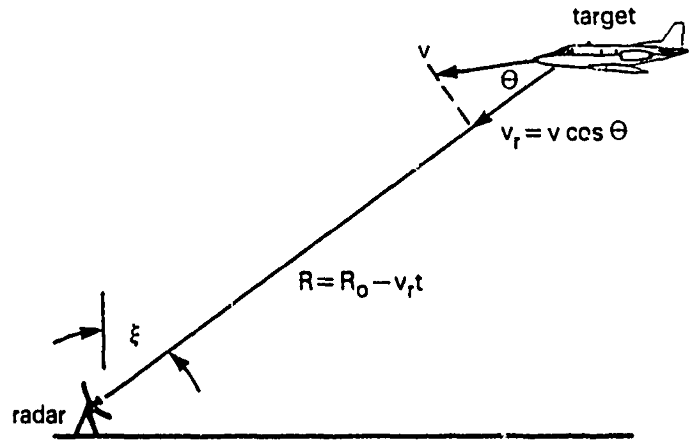

Figure 1 - Radar Illuminating a Moving Target at Range R

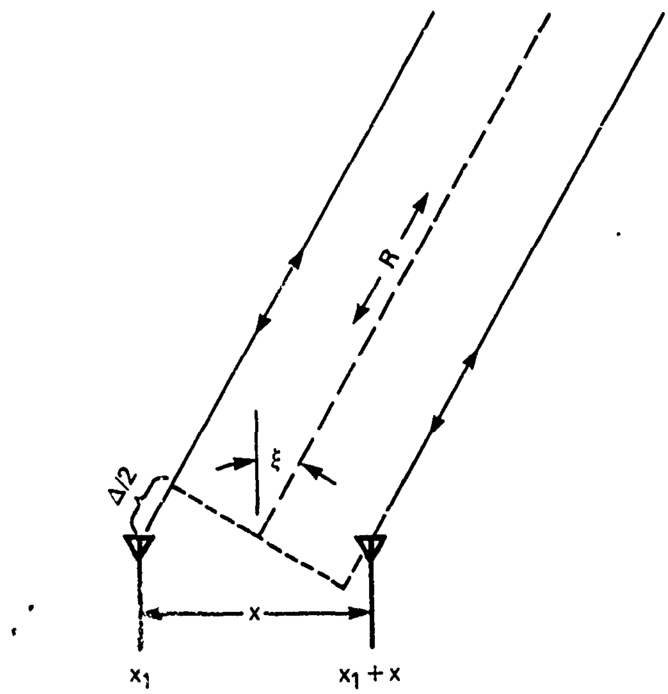

Figure 2 - Simple Interferometer for Measurement of Angle of Arrival ξ; Δ=x sin ξ

velocity of the target, and θ is the angle between the radar line-of-sight and the target's velocity-vector (direction). This assumes the target has a straight-line trajectory with a constant velocity. The radial velocity is $v_r = v \cos \theta$. The phase can be written

$$\phi = 2\pi fT = 2\pi f(2/c)[R_0 - v(\cos \theta)t] \tag{3}$$

Taking the partial derivative of ϕ with respect to time, at a fixed frequency and a fixed position, gives

$$\partial\phi/\partial t|_{f,x} = -4\pi v(\cos \theta)/\lambda = -4\pi v_r/\lambda = -2\pi f_d \tag{4}$$

where f_d is the usual (temporal) doppler frequency shift = $2v_r/\lambda$. (The minus sign in Eq. 4 means the range is decreasing.)

Spatial-position derivative. The measurements of range and range rate are made from a single position of the radar. The measurement of the angle of arrival, however, requires that the phase of the received echo signal be obtained as a function of spatial position. Consider the geometry of Fig. 2 with an antenna at position x_1 that transmits a signal and receives the echo reflected by the target. A second antenna at $x_1 + x$ simultaneously transmits the same signal and receives an echo from the target. The arrangement of Fig. 2 is like that of an interferometer. The signal transmitted at both x_1 and x_1+x is $a_t \sin 2\pi ft$. The received signal at x_1 is

$$s_1(t) = a_r \sin 2\pi f(t - T_1) \tag{5}$$

where $T_1 = (2R + \Delta)/c$, $\Delta = (x/2) \sin \xi$, ξ = angle of arrival. The signal received at position $x_1 + x$ is

$$s_2(t) = a_r \sin 2\pi f(t - T_2) \tag{6}$$

where $T_2 = (2R - \Delta)/c$. This assumes the target is in the far field so that ξ is the same at any point along the baseline. If the two signals of Eqs. 5 and 6 are applied to a phase detector, the output is the phase difference $2\pi f(T_1 - T_2)$, or

$$\phi_s = 2\pi(f/c)\, x \sin \xi = 2\pi(x/\lambda) \sin \xi \tag{7}$$

where $\lambda = c/f$. Dividing ϕ_s by the spacing x between radars (which is known), gives the angle of arrival ξ. More generally, the angle ξ is found from the partial derivative of the phase ϕ_s of Eq. 7 as a function of x, at a particular frequency f and a particular time t; or

$$\partial\phi_s/\partial x|_{f,t} = (2\pi/\lambda) \sin \xi \tag{8}$$

Thus Eq. 8, which represents the variation of phase with spatial position, provides a measurement of the angle of arrival ξ.

The phase ϕ_s of Eq. 7 is a spatial phase that requires observations be made at more than one spatial position. It is different from the temporal phase ϕ given by Eq. 3, which is found from a measurement made at a single point of observation. Lees did not differentiate between these two different phases since he did not express his ideas in mathematical terms. When both

the temporal and spatial phases are combined, as they are in either Eqs. 5 or 6, there can be coupling between the two that makes difficult the extraction of information. This coupling, which is more important when considering the measurement of angle rate to be described later, does not appear in Eq. 7.

It might be noted that the phase given by Eq. 7 can provide the angle ξ if the spacing x is known, without taking a derivative. The measurement based on Eq. 7, however, is highly ambiguous. Ambiguities can be resolved by measuring ϕ_s at more than a single spacing x. This is similar to what is done in obtaining $\partial\phi_s/\partial x$.

A similar result to Eq. 8 can be obtained with only one of the antennas in Fig. 2 transmitting, but with both receiving.

The determination of the angle of arrival by measuring the phase difference between two antennas is not usual in radar. A surveillance radar employs a scanning beam, and the direction the beam points at the time of maximum echo signal strength is taken as the angle at which the target is located. (In a tracking radar, the target's angle location is found from the antenna pointing direction when the difference-pattern echo signal is a minimum.)

A fourth radar measurement (angle rate). The partial derivatives discussed previously provide the three usual radar measurements of range, range rate (radial velocity), and angle. From considerations of symmetry there ought to be a fourth basic measurement -- that of angle rate. Angle rate, of course, can be determined from the rate of change of angle, just as range rate can be found from the rate of change of range rather than from the doppler frequency shift. However, we would like to find the angle rate as a partial derivative of the phase ϕ_s, Eq. 7. It will be seen that the phase derivative that provides the angle rate can be related to a "spatial doppler-frequency shift" just as the range rate can be related to the classical (temporal) doppler-frequency shift.

If the target is in motion, the angle of arrival ξ changes with time. We write $\xi = \xi_1 - (v_c/R)t$, where ξ_1 is the angle at t = 0, $v_c = v \sin\theta$, θ is defined in Fig. 1, v = target velocity, and R = range. The target's angle rate is v_c/R. Assuming $\sin\xi \approx \xi$, the phase of Eq. 7 becomes

$$\phi_s = 2\pi(x/\lambda)[\xi_1 - (v_c/R)t] \tag{9}$$

The angle rate can be written, in general, as the partial derivative of ϕ_s with respect to t, at a fixed frequency and a fixed distance x, or

$$\partial\phi_s/\partial t|_{f,x} = -2\pi(x/\lambda)v_c/R \tag{10}$$

(The wavelength λ is used here rather than the frequency f since it is customary to do so in antenna analysis.) Thus the angle rate v_c/R is found as the rate of change of the spatial phase ϕ_s as a function of time.

The angle rate has not been a major radar measurement in the past, probably for the reason that it is generally a small quantity, except at very short range. However, it has some interesting attributes that make it worth considering.

Spatial doppler-frequency shift. The rate of change of phase with respect to time is a frequency. Thus the partial derivative of ϕ_s (given by Eq. 10) can be thought of as providing a "spatial doppler-frequency shift" equal to $(\partial\phi_s/\partial t)/2\pi$, or (ignoring the minus sign)

$$f_s = \frac{x}{\lambda}\frac{v_c}{R} \tag{11}$$

The spatial doppler frequency might be used, analogous to the usual temporal doppler-frequency shift, to determine the cross-velocity v_c or to separate moving target echoes from fixed target echoes.

Cross velocity, angle rate, and vector velocity. A measurement related to angle rate is the cross velocity, $v_c = v \sin\theta$, where θ was defined in Fig. 1. (The cross velocity is sometimes called the tangential velocity.) The angle rate Ω_t (radians/second) is equal to v_c/R, where R = range. In this discussion, the angle rate and the cross velocity will be considered (almost) synonymous with each other. A method for finding one will be assumed to provide the other if the range is known.

If both the cross velocity v_c and the relative velocity v_r are known, then the vector velocity of the target can be found, since

$$|v| = (v_c^2 + v_r^2)^{1/2} \qquad \text{- speed} \tag{12}$$

$$\theta = \arctan(v_c/v_r) \qquad \text{- direction} \tag{13}$$

Summary of phase-derivative measurements. It has been indicated that there are four basic measurements that can be derived from partial phase derivatives:

$\left.\frac{\partial\phi}{\partial f}\right|_{x,t} \longrightarrow$ range $\qquad$ $\left.\frac{\partial\phi_s}{\partial x}\right|_{f,t} \longrightarrow$ angle

$\left.\frac{\partial\phi}{\partial t}\right|_{f,x} \longrightarrow$ range rate $\qquad$ $\left.\frac{\partial\phi_s}{\partial t}\right|_{f,x} \longrightarrow$ angle rate

The taking of partial derivatives is not the usual radar method for extracting target information, but they can be related to the usual radar measurements (as discussed in Sec. 7).

The interferometer geometry of Fig. 2 can be used to extract the spatial doppler. A signal is transmitted and the echo signals are received by the two antennas separated a distance x. The phase difference ϕ_s between the two received signals is extracted, and the rate of change of phase with time is the spatial doppler-frequency shift. We next consider the use of a scanning interferometer to extract the spatial doppler. In Sec. 5 it is shown that a spatial doppler effect is not restricted to an interferometer antenna, but is also obtained with a single-beam radiation pattern.

4. SCANNING INTERFEROMETER

Description. The classical radar waveform is a sine wave time-domain signal of the form $\sin(2\pi ft)$ where f = frequency. A target approaching the radar with a radial velocity $v_r = v \cos\theta$ traverses an additional v_r/λ cycles per second of the transmitted signal ($\lambda = c/f$). This changes (increases) the apparent frequency seen at the target. The echo signal returned to the radar changes by twice this, or $2v_r/\lambda$, which is the classical doppler frequency shift. We next apply this (overly simplified) reasoning to the spatial domain.

Consider a 2-element interferometer antenna with the elements spaced a distance d apart, as in Fig. 3. The intensity radiation pattern of the interferometer can be shown (Appendix III) to be

$$g(\xi) = 2g_e(\xi)\cos[\pi(d/\lambda)\sin\xi] \tag{14}$$

where ξ is the angle with respect to the normal to the interferometer baseline, and $g_e(\xi)$ is the "element" pattern, or pattern of the individual antennas of the interferometer. The factor 2 in this equation occurs because there are two antennas in the interferometer. It is retained throughout this report. This is arbitrary; it can be readily omitted, if desired, by normalizing the amplitude to unity. If the illumination of the interferometer antennas of dimension D were uniform, $g_e(\xi)$ is $[\sin\pi(D/\lambda)u]/(\pi u)$, where $u = \sin\xi$. (The cosine of Eq. 14 could be replaced with the sine if the two antennas of the interferometer were combined to provide a difference pattern rather than a sum pattern.) It is assumed that the angular extent of the interferometer pattern is limited to the region λ/D by the "element" pattern of the individual interferometer antennas, each of dimension D. For convenience in analysis, the angle λ/D is assumed small so that Eq. 14 can be approximated by

$$g(\xi) = 2g_e(\xi)\cos[\pi(d/\lambda)\xi] \tag{15}$$

A time-varying spatial signal is obtained by scanning (rotating) the interferometer pattern in angle at a rate Ω radians per second. A target at some angle will then see a time-varying cosine spatial waveform extending for a time duration of approximately $\lambda/D\Omega$. (The scanning of the interferometer pattern can be done either electronically or mechanically. Here we assume it is scanned mechanically so as not to be concerned about the change of beam shape that occurs when scanned electronically.) The temporal signal seen at the target will have a carrier of the usual form, $\sin 2\pi ft$, which is modulated (multiplied) by the spatial signal of Eq. 15. The temporal carrier signal at frequency f does not basically affect the spatial waveform, but acts as the means by which the antenna pattern modulation can be utilized. It is the spatial signal generated by the scanning antenna, as given by $g(\xi)$, that is of interest in this discussion. On reception, the carrier represented by $\sin 2\pi ft$ is separated (filtered) from the spatial signal $g(\xi)$.

Scanning the interferometer pattern in angle will result in a time varying signal $g(t)$. Replacing the angle ξ with Ωt, we have as the transmitted

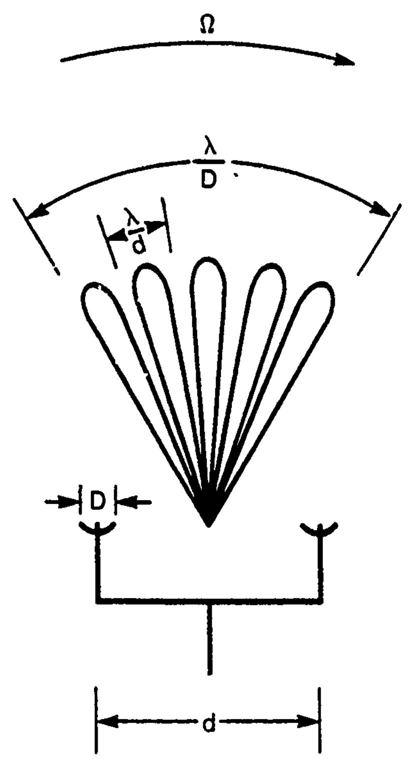

Figure 3 - Radiation Pattern of Two-Element Interferometer

spatial signal

$$g(t) = 2\ \text{rect}(t/T_s)\ \cos[\pi(d/\lambda)\Omega t] \tag{16}$$

where $\text{rect}(t/T_s) = 1$ for $-T_s/2 < t < +T_s/2$, and is zero otherwise. With this notation, the effect of the element pattern is taken into account by assuming the cosine wave of spatial frequency extends over a time duration $T_s = \lambda/D\Omega$. In all that follows the rect function will be omitted as a convenience, but it is always understood that the signals are of finite duration T_s. It is assumed that the cross velocity, $R\Omega$, of the scanning beam is small compared to the velocity of propagation c, so that any effects of the round-trip transit time can be ignored. Also, the antenna scanning rate Ω is assumed large with respect to the target's angular component v_c/R.

The form of the received spatial signal will be the square of that transmitted, or

$$g_r(t) = 4\cos^2[\pi(d/\lambda)\Omega(t-T)] \tag{17}$$

The time T is the time at which the antenna pattern maximum (beam center) strikes the target located at a particular angle ξ_1. ($T = 0$ for a target at $\xi = 0$.) If the target located at angle ξ_1 has no component of angular velocity, the time T is equal to ξ_1/Ω. If the target has an angular component of velocity v_c/R radians/second, the angle ξ is $\xi_1 - (v_c/R)t$, where ξ_1 is the angle of the target at $t = 0$. The spatial signal received from a target with an angular velocity component is then

$$\begin{aligned} g_r(t) &= 4\cos^2\{\pi(d/\lambda)[\Omega t + (v_c/R)t - \xi_1]\} \\ &= 2\{1 + \cos 2\pi(d/\lambda)[\Omega t + (v_c/R)t - \xi_1]\} \end{aligned} \tag{18}$$

The received spatial signal consists of a dc component and an ac component of frequency $d/\lambda(\Omega + v_c/R)$. Its time duration is $\lambda/D\Omega$. The signal received from a stationary target with $v_c = 0$ is

$$s_r(t) = 2[1 + \cos 2\pi(d/\lambda)(\Omega t - \xi_1)] \tag{19}$$

Its spatial frequency is $(d/\lambda)\Omega$. By comparing Eq. 18 and 19, it can be seen that the spatial frequency from the moving target is shifted an amount

$$f_s = (d/\lambda)(v_c/R) \tag{20}$$

which is defined as the <u>spatial doppler-frequency shift</u>.

The dc term in Eq. 18 results from the two-way antenna pattern which causes the gain to appear as the square in the received signal. This is different from temporal signals since a dc component cannot be radiated in space. The spatial signal, on the other hand, can have a dc component. If the signal were transmitted with a broad antenna pattern (as from a single, small antenna) and received with an interferometer pattern, the received signal would be proportional to the square root of Eq. 17 and the spatial doppler frequency would be one-half of Eq. 20. It has no dc term. The transmitted spatial signal of Eq. 16, which is not a squared function, can ha . a dc component depending on the angular extent covered.

Measurement on a single scan. The spatial doppler-frequency shift can be extracted by comparing the spatial frequency from a stationary target with the spatial frequency received from a moving target. The spatial frequency from a stationary target is found from Eq. 19 as $d\Omega/\lambda$. It is known *a priori*.

The measurement of frequency requires time. As an approximate "rule of thumb", the number of cycles in the doppler-shifted signal during the time on target should differ from the number of cycles in the signal from a stationary target (no doppler shift) by at least one in order to determine the spatial doppler shift on the basis of a single scan by the target. Therefore, if the spatial doppler f_s is to be measured on a single scan, we need to have

$$f_s \cdot (\text{time on target}) > 1 \tag{21}$$

The time on target is $\lambda/D\Omega$, where λ/D is the spatial extent of the radiation pattern and D is the dimension of the individual antennas of the interferometer. Thus, it is required that

$$[(d/\lambda)(v_c/R)](\lambda/D\Omega) = (d/D)(v_c/R)/\Omega > 1 \tag{22}$$

If we let d = 10 m, D = 1 m, v_c = 100 m/s, and R = 10 km, then we have

$$(10/1)\ (100/10^4) = 0.1 \text{ rad/s} > \Omega$$

This states that, for this example, the scanning rate Ω must be less than 0.1 rad/s, or 5.7 deg/s, which corresponds to a rotation rate of just under 1 rpm. Thus if an S-band scanning interferometer had its two antennas separated 10 m and had an interferometer pattern 6 degrees wide, the spatial doppler frequency shift of target at a range of 10 km that moved with a cross velocity of 100 m/s (an angular rate of 3.3 milliradian per second) would be detectable on the basis of a single scan of the radiation pattern by the target. However, an antenna rotation rate of 1 rpm and a range of 10 km are small. (In this example, the spatial doppler frequency f_s is 1 Hz and the time on target is one second.) Thus, it might not be usual for the spatial doppler to be extracted on a single scan.

Consider the following (more reasonable) set of parameters with an interferometer having a rotation rate of 10 rpm ($\Omega \approx 1$ rad/s), and with d = 10 m, D = 1 m, v_c = 100 m/s, and R = 100 km. Using the criterion of Eq. 22, we find

$$(d/D)(v_c/R)/\Omega = (10/1)\ (100/10^5) = 0.01$$

which is much less than unity. In this case the spatial doppler shift cannot be measured on a single observation. When it is not practical or possible to obtain a measurable spatial doppler shift on a single scan, multiple scans can be used (analogous to the MTI radar which extracts a doppler frequency shift from sweep to sweep).

Spatial MTI (Moving Target Indication). The received signal is that given by Eq. 18, which is repeated here:

$$s_r(t) = 2\{1 + \cos 2\pi(d/\lambda)[\Omega t + (v_c/R)t - \xi_1]\} \tag{18}$$

It has a time duration $\lambda/D\Omega$. The dc component is removed to give

$$s(t) = 2 \cos 2\pi(d/\lambda)[\Omega t + (v_c/R)t - \xi_1] \tag{23}$$

In the spatial MTI radar (just as in a conventional MTI) the received signal is mixed with a reference, which is twice the scanning spatial frequency, or

$$\text{reference signal} = \cos 2\pi(d/\lambda)\Omega t \tag{24}$$

It is assumed that the reference frequency $d\Omega/\lambda$ is large compared to the spatial doppler frequency, or $\Omega >> v_c/R$. The received signal, Eq. 23, is mixed in a phase detector (or its digital equivalent) with the reference signal, Eq. 24, and the difference is extracted, which is

$$s_1(t) = k \cos[2\pi(d/\lambda)(v_c/R)t - 2\pi(d/\lambda)\xi_1]$$

$$= k \cos[2\pi f_s t - \phi_s] \tag{25}$$

where k is the amplitude (somewhat arbitrary), f_s is the spatial doppler shift = $(d/\lambda)(v_c/R)$, and ϕ_s is the phase $2\pi(d/\lambda)\xi_1$. On the next scan, a time T_s later, the received spatial signal is

$$s_2(t) = k \cos[2\pi f_s(t-T_s) - \phi_s] \tag{26}$$

Subtracting $s_2(t)$ from $s_1(t)$ gives

$$\text{difference} = 2k \sin(\pi f_s T_s) \sin[2\pi f_s(t-T_s/2) - \phi_s] \tag{27}$$

The first sine factor is an amplitude that represents the response of the spatial MTI, analogous to the amplitude response of a MTI delay line canceller. The relative response (normalized by dividing by k) of the "spatial delay line canceller" is then

$$H(f_s) = 2 \sin \pi f_s T_s = 2 \sin \pi(d/\lambda)(v_c/R)T_s \tag{28}$$

This is sketched in Fig. 4.

At those values of f_s which are an integer multiple of $1/T_s$, the response is zero. Thus "spatial blind speeds" v_{cb} occur when

$$(d/\lambda)(v_{cb}/R)T_s = n$$

$$\text{or} \quad v_{cb} = n\lambda R/T_s d \tag{29}$$

With λ = 0.1 m, R = 100 km, T_s = 4 s, and d = 10 m, the first (n = 1) blind speed (cross velocity) is 250 m/s, or about 500 kn. In this case, the spatial blind speeds are higher than the (relatively low) blind speeds in a conventional S-band MTI radar. (With a prf of 400 Hz, the first blind speed of an S-band MTI radar is about 40 kn.) The shorter the range, however, the lower (worse) will be the spatial blind speed.

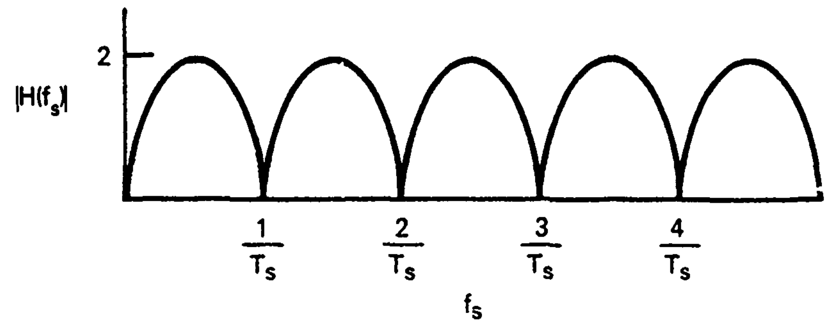

Figure 4 - Sketch of Spatial MTI Response, With T_s=Time Between Observations

The ratio of the spatial blind speed v_{cb} to the radial-velocity blind speed v_{rb} normally obtained in a classical MIT radar is

$$\frac{v_{cb}}{v_{rb}} = \frac{\lambda R}{T_s d} \cdot \frac{2T_p}{\lambda} = \frac{2RT_p}{T_s d} \qquad (30)$$

where f_p = prf = $1/T_p$. Thus there may be situations where the spatial MTI might provide clear operation when the conventional (temporal) MTI experiences blind speeds. Such might be the case where large unambiguous range is desired. Since the unambiguous range R_u is equal to $cT_p/2$, Eq. 30 can be written

$$\frac{v_{cb}}{v_{rb}} = 4R_u^2/cT_s d \qquad (31)$$

(Equation 31 assumes the target is at the maximum unambiguous range.)

The use of spatial MTI with two observations does not provide a measurement of angle rate, but only that the target has a component of angular velocity. Techniques for measuring the angle rate add complication, but are possible. A problem that needs to be examined (but not here) is whether the combining of temporal MTI with spatial MTI is practical.

It was pointed out by Dr. Ben Cantrell (NRL Code 5310) that the degree of clutter cancellation with a spatial MTI could depend on the decorrelation of the clutter over the time T_s; especially since the value of T_s (the antenna scan period) can be considerably greater than the pulse repetition interval (the corresponding observation time in a conventional MTI). In the conventional MTI, clutter movement within the observation time must not be large compared to the carrier wavelength. In the case of spatial MTI, however, the clutter movement must not be large compared to a "spatial wavelength". Thus, clutter decorrelation might not be too severe a problem.

Nonscanning interferometer. If the interferometer is fixed, rather than scanning, the angular (cross-range) component of target velocity and the lobe pattern of the interferometer will cause the received signal to be modulated in amplitude at the spatial doppler frequency $f_s = (d/\lambda)(v_c/R)$. The period of the spatial doppler frequency is the time the target takes in traversing one cycle of a fixed interferometer pattern. With d = 10 m, λ = 0.1 m, v_c = 100 m/s, and R = 100 km, the spatial doppler f_s = 0.1 Hz. Thus the period is 10 s. It is the approximate time duration required to make a measurement of the frequency. (The time can be reduced somewhat if the signal-to-noise ratio is large.) It should be noted that the same time (the reciprocal of f_s) is required in a scanning interferometer as in a fixed interferometer. (The relatively long time required for a measurement is why the spatial MTI processes the signal from scan to scan.) The advantage of scanning the interferometer is not in reducing the time required for a measurement, but in covering a large volume of space with a directive antenna.

5. GAUSSIAN RADIATION PATTERN

In Sec. 4, a cosinusoidal spatial waveform of finite duration was assumed, as is obtained from a two-element interferometer. Similar spatial doppler

effects occur with other spatial waveforms (such as the radiation patterns of conventional antennas).

Consider, for example, a guassian antenna pattern of the form

$$g_t(\alpha) = \exp\left[-2.776\ \alpha^2/\alpha_B^2\right] = \exp\left[-k^2\alpha^2\right] \quad (32)$$

where α_B = half-power beamwidth. Replacing the angle α 1 Ωt to describe the time-varying spatial signal gives

$$g_t(t) = \exp\left[-k^2\Omega^2 t^2\right] \quad (33)$$

The received spatial signal is the two-way signal, with a time delay $T = \xi/\Omega$, or

$$g_r(t) = \exp\left[-2k^2\Omega^2(t-T)^2\right] \quad (34)$$

As before, the received amplitude is taken to be unity. Substituting for $T = \xi/\Omega$ in Eq. 34, where the target angle $\xi = \xi_1 - (v/R)t$, gives

$$g_r(t) = \exp\{-2k^2\ \Omega t - [\xi_1 - (v_c/R)t]\ ^2\} \quad (35a)$$

It will be shown that the half-power width of the received spatial signal is modified by a target with an angular component of velocity. For convenience of analysis, the target will be assumed to be at $\xi_1 = 0$ at $t = 0$. The received spatial signal is then

$$g_r(t) = \exp\{-2k^2\Omega^2 t^2[1 + (v_c/R\Omega)]^2\} \quad (35b)$$

The half-power point t_1 occurs when the argument of the exponential is approximately 0.35. Thus when there is no angular velocity ($v_c = 0$) the the half-power width (in time) $2t_1 = \Delta t_0$, is

$$\Delta t_0 = 2 \times \left[0.35/2k^2\Omega^2\right]^{1/2}$$

$$= (0.84)/k\Omega \quad (36)$$

The half-power width (in time) for a target moving with a velocity v_c is

$$\Delta t = (0.84)/[k\Omega(1+v_c/R\Omega)] \quad (37)$$

The ratio of Eq. 36 to Eq. 37 is

$$\Delta t_0/\Delta t = 1 + v_c/R\Omega \quad (38)$$

If v_c = 100 m/s, R = 100 km and Ω = 1 rad/s, the ratio of widths of the transmitted and received signals is 1.001, or a change of 0.1%. This is likely to be difficult to measure. If, on the other hand R = 10 km and Ω = 0.1 rad/s, the ratio of widths is 1.1, a 10% change, which ought to be much easier to detect.

When using the spatial radiation pattern it is assumed that there is a substantial number of pulses received within the pattern and that the amplitude fluctuations of the echo signals do not affect the accuracy of the

spatial pattern measurements (which follows from the basic assumption of this section that the target is considered to be a point scatterer).

6. "MATCHED FILTER" DETECTION OF SPATIAL SIGNALS

We return to the scanning interferometer, as discussed in Sec. 4, to examine the processing of a spatial signal. "Matched filtering" is considered here in both the temporal and spatial domains. The matched filter is one which maximizes the peak-signal-to-mean-noise ratio and has a frequency response H(f) proportional to the conjugate of the Fourier transform of the signal to be detected, or $S^*(f)$. The matched filter in the temporal domain will be considered first. (This is not the matched filter for temporal signals, but the temporal filter used for extracting spatial signals.)

Spatial matched filter in the time domain. The received signal is given by Eq. 18, which is repeated below:

$$g_r(t) = 2\{1 + \cos 2\pi(d/\lambda)[\Omega t + (v_c/R)t - \xi_1]\} \tag{18}$$

where d = interferometer antenna separation, λ = wavelength, Ω = antenna scanning rate, v_c = cross velocity, R = range, t = time, and ξ_1 = angle of target at t = 0. The signal of Eq. 18 has a time duration $\lambda/D\Omega$, where D = diameter of an individual antenna. If the cross velocity $v_c = 0$, Eq. 18 becomes

$$g_r(t) = 2\{1 + \cos 2\pi(d/\lambda)[\Omega t - \xi_1]\} \tag{39}$$

The received signal has a dc component and an ac component with twice the spatial frequency of the transmitted signal (Eq.16). The spectrum of the received signal $g_r(t)$ can be shown to be

$$\begin{aligned} S_r(f) &= \frac{2 \sin(\pi f\lambda/D\Omega)}{\pi f} \\ &- e^{+j2\pi(d/\lambda)\xi_1} \frac{\sin \pi[(d/\lambda)\Omega+f](\lambda/D\Omega)}{\pi[(d/\lambda)\Omega+f]} \\ &+ e^{-j2\pi(d/\lambda)\xi_1} \frac{\sin \pi[(d/\lambda)\Omega-f](\lambda/D\Omega)}{\pi[(d/\lambda)\Omega-f]} \end{aligned} \tag{40}$$

The exponential factors represent fixed phase shifts. The angle ξ_1 can be set to zero without loss of generality. (ξ_1 is a known function of time since the scanning rate Ω is known.) There are three terms: a dc component, and components at $f = \pm (d/\lambda)\Omega$, Fig. 5a. (Only the positive frequencies are shown. The dc component has its first zero at $f = D\Omega/\lambda$. The ac component is centered at $f = \pm d\Omega/\lambda$ and has a null-width of $2D\Omega/\lambda$. (The sidelobes of the sin X/X frequency response are not shown, so as to keep the diagram uncomplicated.) The matched filter for the received signal has a frequency response equal to $S^*(f)$. For a symmetrical signal, as in Fig. 5a, this is simply $S_r(f)$, the solid curve.

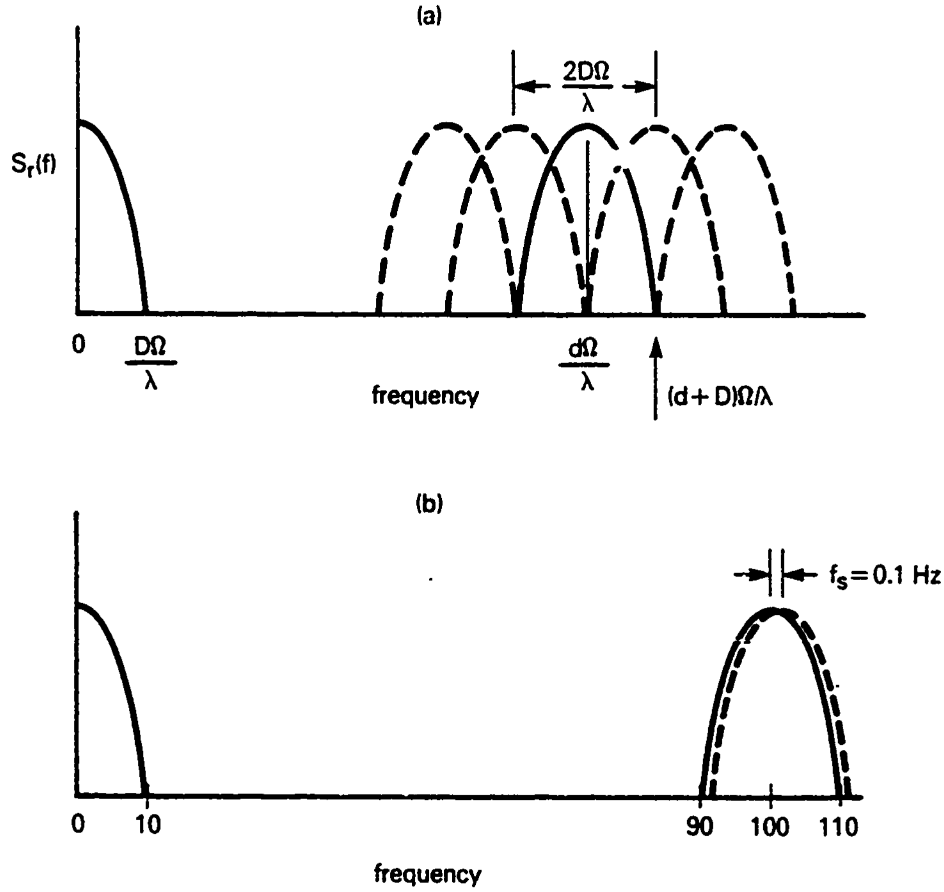

Figure 5 - (a) Spatial matched filter response (spectrum of the received spatial signal, Eq. 40); solid curve applies for v_c=0; dashed curves represent filters in a spatial matched filter bank.
(b) Example of spatial doppler frequency response of a moving scatterer (dashed curve) compared to a non-moving scatterer (solid curve), as described in the text following Eq. 41.

The above applies to a target with zero cross-velocity; i.e., $v_c = 0$. From Eq. 18, a target with an angular velocity will have its response centered at a frequency $d\Omega/\lambda + (d/\lambda)v_c/R$, where the second term is the spatial doppler $f_s = (d/\lambda)(v_c/R)$. The classical approach for detection when the doppler frequency is unknown is to use a bank of matched filters. The individual filters of the analogous spatial matched filter would have frequencies centered at $d\Omega/\lambda \pm nD\Omega/\lambda$, where $n = 0, 1, 2,...N$. The integer N depends on the maximum doppler to be expected. The received signal, Eq. 18, will be beyond the half-power width of the filter matched to zero velocity if

$$f_s = (d/\lambda)(v_c/R) > D\Omega/2\lambda$$

$$\text{or} \quad v_c > (DR\Omega)/2d \tag{41}$$

If, for example, D = 1 m, R = 100 km, Ω = 1 r/s, and d = 10 m, then $v_c > 5000$ m/s before a filter bank is needed. With such numbers, a filter bank would not be required, and only a single filter could be used. Using the example numbers usually selected in this report to illustrate the effects, (d = 10m, D = 1m, Ω = 1 m/s), the spatial scanning frequency dD/λ is 100 Hz and the spectral width between zeros is $2D\Omega/\lambda$ = 20 Hz. The spatial doppler frequency with v_c = 100 m/s and R = 100 km is f_s = 0.1 Hz. This is illustrated in Fig. 5b. Thus, in this case, the spatial doppler frequency is well within the pass band of the zero-doppler filter. If, however, the interferometer did not scan, the processing has to be modified and a "bank" of filters then might be required.

Spatial matched filter in the aperture domain. In the above it was indicated that matched filtering of the spatial signal in the time domain should be adequate. However, it is of interest to determine what spatial matched filtering might be like as an antenna problem.

The received signal with a spatial doppler-frequency shift $f_s = (d/\lambda)(v_c/R)$ is written as

$$g_r(t) = 2\{1 + \cos(2\pi[(d\Omega/\lambda) + f_s]t - \phi_1)\} \tag{42}$$

where $\phi_1 = 2\pi(d/\lambda)\xi_1$. The signal is assumed to be transmitted by a two-element inteferometer with antenna separation equal to d. If the spatial doppler shift f_s is sufficiently large, the received signal is not "matched" to the interferometer of spacing d, but to some other interferometer spacing d', given by

$$d'\Omega/\lambda = d\Omega/\lambda + f_s$$

$$\text{or} \tag{43}$$

$$d' = d + f_s\lambda/\Omega$$

There is a minimum value of d' - d = 2D since the two interferometers cannot be spaced closer to one another without the antennas overlapping, unless a phased array is used. The geometry is shown in Fig. 6. The echo signal

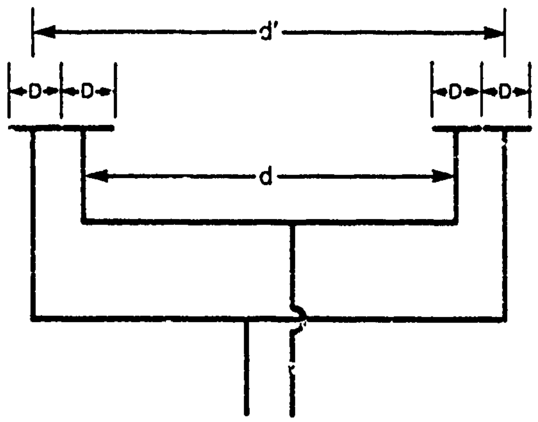

Figure 6 - Geometry of Two Interferometers of Spacing d and d'=d+2D

will be matched to the interferometer of spacing d' rather than d if

$$d' = d + (f_s\lambda)/\Omega > d + 2D \tag{44}$$

or $f_s\lambda/\Omega > 2D$

Substituting for f_s

$$v_c > 2\,\Omega(D/d)R \tag{45}$$

With $\Omega = 1$ r/s, $D/d = 0.1$, and $R = 100$ km, v_c must be greater than 20,000 m/s before a separate receiving interferometer is needed. This velocity is quite large. (At 10 km, the velocity must be greater than 2000 m/s, which is also large.) Thus it is not likely under most circumstances that a spatial matched filter bank will be required with a scanning interferometer.

7. RELATION TO CONVENTIONAL RADAR MEASUREMENTS

The phase derivatives for measuring target location and velocity differ from the conventional measurements made by a radar. This section discusses the relationship between the phase-derivative method and the conventional method. Emphasis is placed on the range measurement, but similar arguments apply as well to the other radar measurements.

Range measurement -- from two frequency components to many. The measurement of range from the partial derivative of phase with respect to frequency, as is discussed in this report, is not how range is determined by conventional radar. Range is usually measured from the time taken by a waveform (such as a short pulse) to transit to the target and back. It is well known, that the signal bandwidth determines the accuracy of the conventional range measurement. (The greater the bandwidth of the radar signal the more accurate will be the range.) The measurement of range as a partial derivative with respect to frequency also involves the frequency domain. It is of interest to examine how the conventional measurement of range as a time delay is related to the measurement of range as a partial derivative of phase with respect to frequency.

The simplest implementation of the phase derivative measurement of range is the two-frequency technique in which range is obtained from the difference in phase of two closely spaced frequencies. At a frequency f_1 the phase from a target at range R is $\phi_1 = 4\pi f_1 R/c$, where c = velocity of propagation. The phase from a signal at frequency f_2 is $\phi_2 = 4\pi f_2 R/c$. The difference $\phi_2 - \phi_1$ between these two phase measurements gives the range

$$R = (c\Delta\phi)(4\pi\Delta f) \tag{46}$$

where $\Delta\phi = \phi_2 - \phi_1$ and $\Delta f = f_2 - f_1$. This technique for determining range is not widely used in radar; however, it has been known for a long time and has seen some application. It is the basis for the multiple-CW-frequency technique used for measuring distance in surveying and for range instrumentation. It is also related to the so-called "delta-k" radar used in remote sensing.

There are two limitations to the range measurement based on the phase-difference between two frequencies: (1) its accuracy is low since the frequency difference Δf must be small in order to avoid ambiguities (this results from the requirement that $\Delta\phi$ must be less than 2π radians) and (2) it cannot provide a range measurement when more than one scatterer is present within the radar resolution cell (since the phase difference $\Delta\phi$ cannot be extracted when unresolved echo signals from multiple scatterers are present.) Both of these limitations will be examined.

The accuracy of the range measurement of a single scatterer can be improved by increasing the frequency separation Δf. It can be shown (from Eq. 24 of Appendix VI.) that the theoretical rms error in measuring range with two frequencies is

$$\delta R = \frac{c}{2\pi \; \Delta f \; (2E/N_0)^{1/2}} \tag{47}$$

where Δf is taken as the bandwidth B, E = received signal energy and N_0 = noise power per unit bandwidth. The maximum unambiguous range occurs when $\Delta\phi$ (in Eq. 46) is equal to 2π radians, or

$$R_{un} = c/(2\Delta f) \tag{48}$$

For a given unambiguous range, the accuracy cannot be improved without limit (by making Δf large in Eq. 47) if range ambiguities (as given by Eq. 48) must be avoided. It is possible, however, to make accurate, unambiguous measurements of range on a single target by using more than two frequencies, Fig. 7. If n frequency components are employed, the maximum spacing ($f_n - f_1$) determines the range accuracy. This frequency difference, however, usually gives a highly ambiguous range measurement. The next-to-the-largest spacing ($f_{n-1} - f_1$) is chosen to resolve the ambiguities in the accurate but ambigambiguous measurement made with f_n and f_1. If the frequency separation $f_{n-1} - f_1$ cannot be made small enough to completely remove the ambiguities, another frequency pair (f_{n-2}, f_1) has to be used. Additional frequencies with closer spacings are added until the separation $f_2 - f_1$ provides the required unambiguous range. Thus the frequencies are chosen so that the widest spacing gives the accuracy required (Eq. 47), the closest spacing gives the unambiguous range (Eq. 48), and the intermediate spacings resolve the ambiguities. Radars that have used this measurement technique have typically employed about five frequencies with separations in the ratio of 10 to 1.

The above has described the measurement of range in the frequency domain. There is also an equivalent time-domain representation that can be found from the application of the Fourier transform. What can be done in the frequency domain has a counter-part in the time domain. The physical implementation might be different, but the results should be the same. (A well-known example from signal detection theory is the frequency domain matched filter and the time domain correlation receiver, both of which maximize the peak-signal-to-mean-noise ratio, but with considerably different physical implementations. It is sometimes instructive to examine a frequency-domain technique in the time domain, and vice versa. (The time domain range measurement technique assumes all frequencies are radiated simultaneously. This need

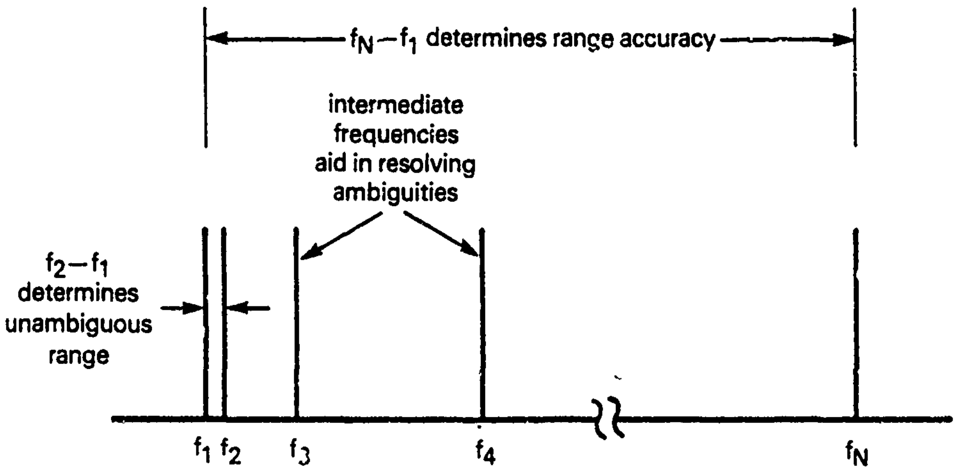

Figure 7 - Accurate, Unambiguous Range Measurement From the Phase-Differences Between Multiple Frequency Components

not be so in the frequency domain if the scatterer is stationary.) The multiple-frequency range measurement will be examined in the time domain.

On the right-hand side of Fig. 8a is a sketch of the envelope of the temporal waveform produced by the sum of two frequencies

$$\sin 2\pi f_1 t + \sin 2\pi f_2 t = 2 \cos\pi\Delta f t \sin(2\pi f_m t) \tag{49}$$

where $\Delta f = f_2 - f_1$ and $f_m = (f_2 + f_1)/2$. The magnitude of the envelope is $|2 \cos(\pi\Delta f t)|$. The carrier at the average frequency $(f_2 + f_1)/2$ is not shown, so as to keep the figures simple. Although the waveform of Fig. 8a and Eq. 49 is not the typical pulse-train waveform usually associated with radar, it might be considered as a special case where the pulse is broad in width with a cosine rather than rectangular shape. Its null width is $1/\Delta f$ (half-power width = $1/2\Delta f$) and the pulse repetition frequency is Δf; hence the duty cycle is unity. The range to a scatterer can be found, in principle, by determining the time at which the cosine "pulse" arrives at the radar. The accuracy will be poor because the width of the "pulse" is large. The large width of the "pulse" also makes it unlikely that the range to multiple targets can be determined. The accuracy of the measurement can be improved by increasing Δf, but this only increases the ambiguity problem.

The range accuracy can be improved with the same unambiguous range (same value of Δf) if a third frequency f_3 is added such that $f_3 - f_1$, is much larger than $\Delta f = f_2 - f_1$. More than three frequencies might be used. A sketch of the time variation of the sum of four frequencies is shown on the the right-hand side of Fig. 8b. (Only the envelope is shown.) The unambiguous range interval is determined, as before, by the smallest frequency separation Δf, and the width of the major response (which determines the accuracy) is inversely proportional to the widest frequency separation. This example shows qualitatively how the addition of a few frequencies improves the range measurement accuracy (because of the narrowing of the time waveforms) while maintaining the unambiguous range. However, the time-sidelobes of this waveform are poor so that it does not perform well if more than one target echo is present.

An accurate range measurement of more than one scatterer within a specified unambiguous range interval requires a "filled" rather than the "thinned" spectrum of Fig. 8b. There needs to be many frequencies with equal spacing Δf covering a bandwidth B, as illustrated in Fig. 8c. Range Range accuracy is determined by the bandwidth B, and the unambiguous range by Δf. The ability to resolve multiple scatterers, and measure the range of each, depends on having the spectrum filled. In principle, there needs to be $N = (B/\Delta f) + 1$ frequency components if N scatterers are to be resolved. From the above, and Fig. 8c, N - 1 equally spaced, equal-amplitude scatterers can be resolved with N equally spaced, equal amplitude frequency components. In practice, the amplitudes of the frequency components are not equal since this results in a waveform with high time sidelobes. Lower time-sidelobes are obtained by tapering the frequency spectrum so that the frequency components in the center are larger than those at the edge, just as is done with the antenna aperture illumination to achieve low sidelobes. The tapered

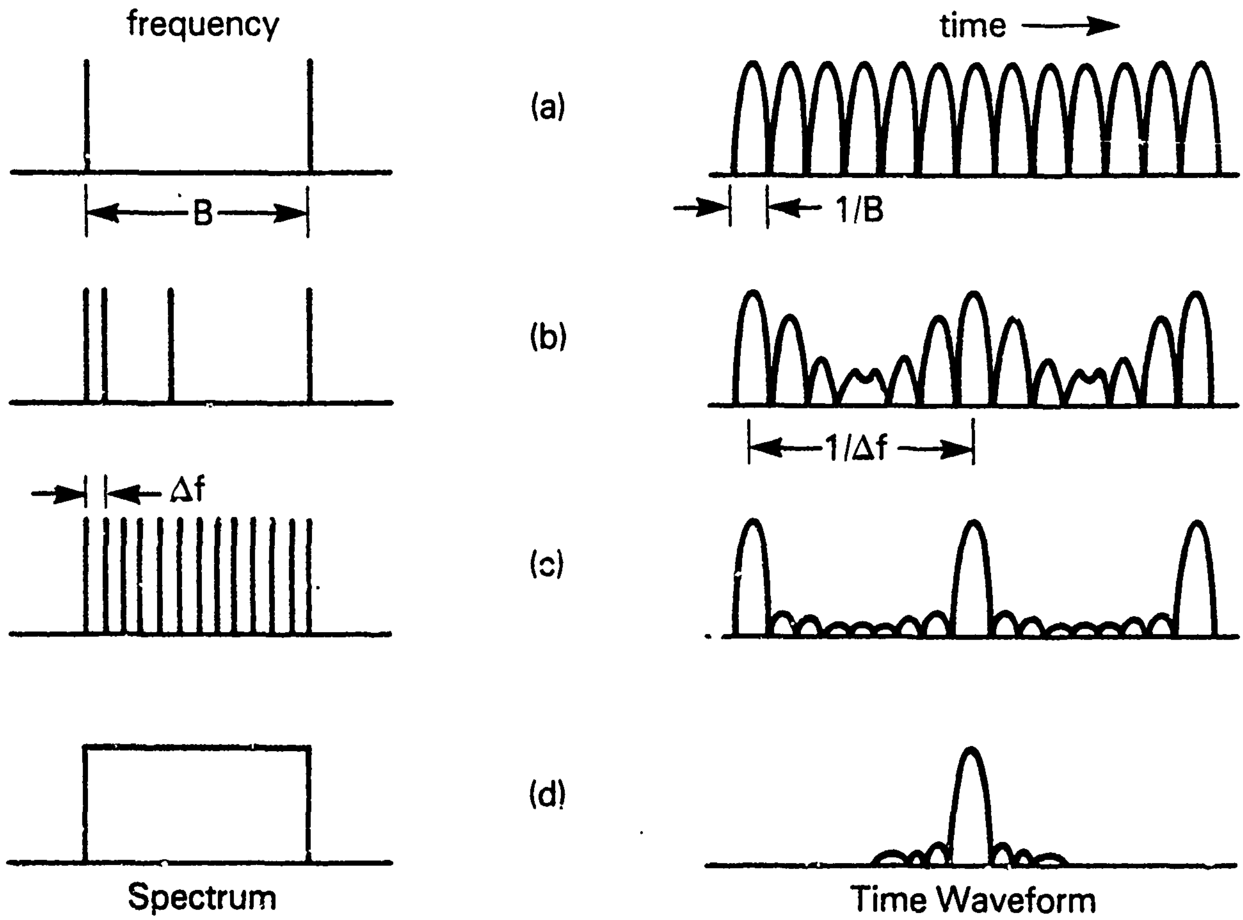

Figure 8 - Spectra (left-hand side) and the envelope of the corresponding time waveform (right-hand side). (a) Two CW frequencies providing inaccurate and/or ambiguous range measurement. (b) Four frequencies arranged to produce an accurate range measurement of a single scatterer within the unambiguous interval. (c) Uniformly filled line-spectrum producing a periodic waveform for accurately measuring the range of multiple targets within the unambiguous interval. (d) Continuous spectrum producing a waveform capable of accurate, unambiguous range measurement of multiple scatterers.

"filled" line spectrum produces a repetitive train of "pulses," the type of radar waveform commonly used in radar for range measurement. As the number of frequencies is increased with a fixed bandwidth B, $N \to \infty$, $\Delta f \to 0$, and $N\Delta f \to B$, there results a continuous spectrum that generates a single pulse with a time duration of about 1/B, as in Fig. 8d. This demonstrates the difference between a filled, continuous spectrum (Fig. 8d), that has no ambiguities, and a discrete spectrum (Fig. 8a) that can have many ambiguities.

In the above, we have shown how to go from the two-frequency radar where a phase-difference measurement gives an unambiguous, but inaccurate, range measurement of a single scatterer, to the conventional pulse waveform capable of measuring the range of multiple targets.

The determination of range from the difference in phase $\Delta\phi$ between two closely spaced frequencies Δf is an embodiment of the phase-derivative method of measuring range. A major difference between the simple phase-derivative method and the usual measurement of range with a short pulse is that a single phase measurement has meaning only for a single scatterer. Thus the two-frequency method does not provide the ranges of multiple scatterers as can the conventional pulse waveform. When there are multiple "point" scatterers within the radar resolution cell, the phase extracted from the echo signal is a composite of the contributions from each scatterer. With N scatterers, each producing an echo signal of amplitude a_i and phase ϕ_i (where i = 1 to N, $\phi_i = 2\pi f T_i$, and T_i = round trip time delay to the ith scatterer), the target echo signal can be represented as

$$\Sigma a_i \sin(2\pi ft + \phi_i) = A \sin(2\pi ft + \Phi) \qquad (50)$$

The phase Φ is given by the classical expression

$$\Phi = \arctan \frac{\sum_{i=1}^{N} a_i \sin \phi_i}{\sum_{i=1}^{N} a_i \cos \phi_i} \qquad (51)$$

A single measurement of phase ϕ (at a single frequency f) as given by Eq. 51 does not provide information about the N individual scatterers. The breakdown of the phase-derivative measurement when multiple target echoes are present is found to be similar to the problem of angle or range glint in a tracking radar. A tracking radar is designed to operate with the phase from a single scatterer only. When multiple scatterers are present within its resolution cell, the angle (or range) measurement can be misleading. This is called glint. It can be severe enough to even cause radar tracking to be directed outside the physical extent of the target. Similarly, the measurement of range from the rate of change of phase with respect to frequency assumes that only a single "point" scatterer is present. With multiple scatterers, the apparent time delay (range) given by $\partial\phi/\phi f$ can also be outside the range extent of the scatterers.

As an example of the problem that occurs when more than one scatterer is within the radar resolution cell, we take the partial derivative with

respect to frequency of the phase given by Eq 51, which is

$$\frac{\partial\Phi}{\partial f} = \frac{\sum\limits_{i}\sum\limits_{j} 2\pi T_i \; a_i a_j \cos 2\pi f(T_i - T_j)}{\sum\limits_{i}\sum\limits_{j} a_i a_j \cos 2\pi f(T_i - T_j)}$$

$$= 2\pi T_e \tag{52}$$

where T_e is an "effective" time delay, and is defined from the above equation. Take the simple case of two scatterers with $a_1 = a$, $a_2 = \rho a$, ($\rho<1$), $T_m = (T_1+T_2)/2$, and $\Delta T = (T_2-T_1)/2$. Then

$$T_e = T_m + \frac{1 - \rho^2}{1 + \rho^2 + 2\rho \cos(2\pi f\Delta T)} \tag{53}$$

When the amplitudes of the two signals are equal ($\rho = 1$), then $T_e = T_m$, where T_m is the midpoint between the two scatterers. This is a "satisfying" answer since it seems to indicate that the effective time delay T_e might be some sort of weighted time delay or target "center of mass." Unfortunately, this is not so. When $\cos 2\pi f\Delta T = -1$, the second term of Eq 53 blows up (equals infinity). The effective time delay T_e as given by Eq. 52 and 53, therefore, is not a usable measure of a target "center of mass." The second term of Eq. 53 is similar in form to that in the classical expression describing the phenomenon of glint in a tracking radar.

Thus the single phase-derivative method breaks down when multiple scatterers are present within the resolution cell of the radar.

A note of caution (and hope), however, should be stated regarding the above. When Eq. 53 blows up and gives a value of T_e that is not related to reality, the amplitude of the received signal goes to zero. Consider the case where $a_1 = a_2$ and the relative phase between the two signals is π radians. Then the resulting signal amplitude is zero and $T_e = \infty$; i.e., the two signals cancel. T_e might still be a useful measure of time delay to a distributed target (a collection of scatterers) if the measurement is ignored when the amplitude is too low.

In principle, the measurement of the composite phase Φ and composite amplitude A of the echo signals from N frequency components can provide the range to N scatterers. The composite amplitude A in Eq. 50 (the companion equation to the phase of Eq. 51) is

$$A^2 = (\Sigma a_i \sin 2\pi f T_i)^2 + (\Sigma a_i \cos 2\pi f T_i)^2 \tag{54}$$

Thus if the composite phase (Eq 51) and amplitude (Eq 54) are measured at N frequencies there are enough equations to find the N values of a_i and the N values of ϕ_i. The N values of ϕ_i provide the ranges to N scatterers. (This can be a formidable task.)

We again examine the frequency-domain representation. The temporal response from a collection of point scatterers at various ranges $cT_i/2$, $i = 1, \ldots, N$, (c = velocity of propagation) is

$$s_r(t) = \sum_i a_i s(t - T_i) \tag{55}$$

where s(t) represents the transmitted signal, a_i is the amplitude of the echo signal from the ith point scatterer, and T_i is the time delay for each scatterer. If the Fourier transform of s(t) is S(f), then the Fourier transform of $s(t - T_i)$ is given by the shifting theorem as $S(f) \exp[-j2\pi fT_i]$. The Fourier transform of the received signal (Eq. 55) is

$$\begin{aligned} S_r(f) &= \sum_i a_i S(f) \exp[-j2\pi fT_i] \\ &= S(f) \sum_i a_i \exp[-j2\pi fT] \end{aligned} \tag{56}$$

If there are N equal-spaced, equal-amplitude scatterers ($a_i = 1$) extending from T_1 to $T_1 + (N-1)\Delta T$, the received spectrum is

$$S_r(f) = S(f) \frac{\sin \pi fN\Delta T}{\sin \pi f\Delta T} \exp j[-2\pi fT_1 - \pi f(N-1)\Delta T] \tag{57}$$

The magnitude (amplitude) of the spectrum $S_r(f)$ is

$$|S_r(f)| = |S(f) (\sin \pi fN\Delta T)/(\sin \pi f\Delta T)| \tag{58}$$

Since the transmitted spectrum S(f) is known, the sin NX/sin X factor will provide the extent of the target, $N \Delta T$. The spacing between the lobes is $1/\Delta T$, which gives the spacing between the equally spaced scatterers. The phase ϕ_r of the spectrum, as given by the exponential of Eq. 57, is

$$\phi_r = -j2\pi fT_1 - \pi f(N-1)\Delta T \tag{59}$$

The partial of ϕ_r with respect to frequency is

$$\partial\phi_r/\partial f = -2\pi[T_1 + (N-1)\Delta T/2] = 2\pi Te \tag{60}$$

which is the time delay (range) to the center of the N equal-amplitude, equally spaced scatterers. Although this result produces a seemingly satisfactory answer, similar catastrophic events can occur as was the case for the two-scatterer target (Eq. 53) discussed previously in the time domain.

Other measurements. The above discussion of the measurement of range can be directly applied to the measurement of angle, starting with a two-element array antenna (interferometer), progressing to a thinned array antenna with several unequally spaced elements, and to a filled phased array without grating lobes (no angle ambiguities), and to a continuous aperture antenna. Nothing further need be said in the section regarding the angle measurement.

A doppler-frequency measurement (range rate, or radial velocity) can be made from the phases obtained at two separate times, analogous to the measurement of the phases at two frequencies to provide range. An MTI radar makes this type of measurement in order to separate moving targets from stationary clutter. The doppler-frequency measurement in an MTI radar, however, is usually highly ambiguous since the pulse repetition period (time between phase samples) is made long to avoid range ambiguities (rather than short to avoid doppler ambiguities.

The doppler ambiguities in an MTI radar result in ambiguities in radial velocity that are called "blind speeds." A target with a radial velocity equal to a blind speed is attenuated and not seen. The usual method to mitigate the effect of blind speeds is to employ more than one pulse repetition frequency (prf) so that the target will be outside a blind speed on at least one prf. Although the MTI radar does not need to know unambiguously the doppler frequency, or radial velocity, it is possible to do so based on the above discussion of the selection of frequencies for making an accurate, unambiguous measurement of range. This is done in Appendix IV.

A brief discussion of phase derivative measurements compared to other radar measurements is given in Appendix V.

Measurement accuracy. Appendix VI examines the accuracy of the phase derivative measurement methods and compares them with the accuracy of conventional radar measurement methods. The following are demonstrated in Appendix VI:

- The theoretical rms error in measuring the radial velocity from either the doppler frequency shift or the rate of change of phase with frequency ($\partial\phi/\partial f$) is approximately B/f_0 times the rms error obtained from a from a measurement of the rate of change of range with time, where f_0 is the carrier frequency and B is the bandwidth of the signal. (Typical ratios of f_0/B might be from 30 to 300.)

- Likewise, the rms error in measuring angle rate with a scanning interferometer or from the rate of change of phase with spatial position is D/d times the rms error obtained from the classical measurement of the rate of change of phase with time, where D is the dimension of the individual interferometer antennas and d is the spacing between the two antennas (each of dimension D) of the interferometer. (The ratio d/D might typically be from about 10 to perhaps 100.)

Thus range rate and angle rate measurements based on the partial derivatives of phase or the doppler frequency shift (spatial as well as temporal) can be much more accurate than measurements based on the first derivative of range or angle with respect to time.

Appendix VI also shows that the accuracy of the measurement of range or angle based on partial phase derivatives is about the same as can be obtained with conventional range or angle measurement.

Appendix I - Review of Related Literature

This appendix briefly reviews the highlights of several publications that relate to the subject of this report. Its purpose is to provide a brief historical background and to place the work reported in this report in proper perspective with respect to other work. The first paper is that of R. J. Lees, which was mentioned in the main body of the report. The other two publications are related to the subject matter of this report, but not in as direct a manner as that of Lees.

I. 1. "A Generalized Theory of Radar Observations," by R. J. Lees, AVIONICS RESEARCH: SATELLITES AND PROBLEMS OF LONG RANGE DETECTION AND TRACKING, AGARDograph 40, Pergamon Press, N.Y. 1960.

This was the first paper to describe radar measurements in terms of the phase and amplitude spectra of the echo signal. It is a short paper (about four pages) and does not go into any detail. All of his discussion and examples are qualitative. No mathematical analysis is given.

He states that the measurment of the amplitude or phase at a single frequency, time, or position provides little information about a target, other than something is present that can cause an echo. It is changes in these quantities that are needed for extracting target information.

The main body of the present report includes several extensions and corrections to Lees's concept. These include the following:

- Lees correctly mentions that the phase derivatives with respect to frequency, time, and position provide the range, radial velocity, and angle of point scatterers; but he also mentions that the amplitude " derivatives" of the same parameters provide information about finite size targets. However, it is not the amplitude derivatives that provide target information, but the variation of the amplitude as a function of these parameters. (Lees does use the term amplitude variations, but he seems to mean the derivatives.) The amplitude derivatives of a distributed scatterer are not constant, as are the phase derivatives that apply for a point target.

- He states that the phase derivatives provide information about a "point" scatterer (he used the term "source") but he did not indicate that the point scatterer had to be the only scatterer present; (or else it has to be resolved in some manner from other scatterers.)

- Lees does not indicate there is a fourth basic phase measurement that gives the angle rate and that there is a fourth basic amplitude measurement that provides the change of cross-range profile with time.

- What Lees calls "shape" is actually the radial profile and what he calls "change of shape" or "target spin" is actually the change of the radial profile with time.

Lees also briefly mentions polarization as providing a measure of target asymmetry, and that ambiguities in measurements will result if the observation is not continuous.

I. 2. "A Radar Tangential Moving Target Indicator," by G. O. Young, Proc. 1984 IEEE National Radar Conference, Atlanta, Ga., pp 90-94.

This paper is concerned with the measurement of target tangential velocity, or angle rate, as well as the detection of crossing targets that would normally be suppressed by conventional MTI processing. Dr. Young does not cite any prior references, but the wording he uses in parts of his paper indicates he is probably familiar with either Lees's original paper or the description of it in Chap. 10 of the first edition of "Introduction to Radar Systems." It is a qualitative paper in that it presents ideas without also justifying that they can be achieved with a practical radar implementation. He does not give quantitative (numerical) examples to show that these concepts can be applied.

He offers two methods for finding the angle rate. In his first method he establishes four equations that relate four radar measurements to four target parameters. The four measurements are $\partial\phi/\partial f$, $\partial\phi/\partial t$, $\partial\phi/\partial x$, $\partial^2\phi/\partial t\partial x$, where ϕ is not the same as that used in this report but includes both the range component (as given by Eq 3 in the main body of this report) and the spatial phase (as given by Eq 7). The target parameters to be found are R_0, R_0', θ_0, and θ_0', where the subscript zero is the value at t = 0. He finds θ_0', as well as the other three parameters, by solving the four equations simultaneously. He gives no indication how realistic this might be in practice, nor does he discuss the value of this approach, nor its advantage compared with other methods for extracting angle rate.

The other method for obtaining angle rate involves taking the difference between the doppler frequencies at two separated antennas. (This is sometimes called differential doppler.) He points out that for the spatial doppler frequency (which he calls the tangential doppler frequency) to be comparable to the values obtained for the usual (temporal) doppler, the spacing between antennas should be comparable to the range. For this reason he states that the ranges must be small if the antenna separation is not to be impracticably large. He states that the technique is well suited to synthetic aperture radar (SAR), but he does not elaborate - and it is not obvious - why this might be true, other than the large separations possible with a moving vehicle carrying a SAR.

In the last paragraph of the paper the use of the interferometer is suggested, but no details are given. The effect on the angle-rate measurement of scanning the radiation pattern also is not mentioned. He seems to require that the target move through the beams, rather than scan the beams through the target.

A possible limitation to his method is that he has to eliminate the constant phase term $4\pi fR_0/c$ from the argument of a sine function representing the received signal. This he claims can be done from the conventional measurement of range (determined from the range gate in which the received pulse lies).

This does not seem practical since the accuracy of the phase $4\pi fR_0/c$ must be within a small fraction of 2π radians in order to subtract the phase term in the sinusoid. This means that the range must be known to a small fraction of a wavelength, which is not likely.

I. 3. "Some Results From Utilizing Doppler Derivatives," by N. Levanon, IEEE Trans., vol AES-16, pp 727-729, September 1980.

Assuming a straight-line target trajectory at constant velocity, the author shows that the range, range-rate, and angle at t = 0 can be found from the measurement of doppler frequency and its first two derivatives. The three equations for f_d, f_d' and f_d'' are solved simultaneously. Alternatively, the range, range-rate, and angle can be found from measurements of f_d at three or more times. No mention is made of angle rate.

Appendix II - Phase Spectrum from a Point Scatterer

The transmitted signal can be represented as having an amplitude a(t) and a phase $\phi(t)$ modulating a carrier frequency f_0, or

$$s_t(t) = a(t) \sin[2\pi f_0 t + \phi(t)] \tag{1}$$

It has a Fourier transform given by

$$S_t(f) = \int_{-\infty}^{\infty} a(t) \sin[2\pi f_0 t + \phi(t)]\, e^{-j2\pi ft}\, dt \tag{2}$$

The signal received from a point scatterer is of the same form as Eq 1, but it is reduced in amplitude by an amount ρ ($\rho<1$) and has a time delay T, so that

$$s_r(t) = \rho s_t(t-T) \tag{3}$$

For convenience, the scale factor ρ is set equal to unity. The spectrum of of the received signal is given by the Fourier shifting thaorem as

$$S_r(f) = S_t(f)\, e^{-j2\pi fT} \tag{4}$$

Normalizing the received spectrum $S_r(f)$ by that which was transmitted $S_t(f)$ gives

$$S_r(f)/S_t(f) = S_0(f) = e^{-j2\pi fT} \tag{5}$$

Thus, the normalized spectrum $S_0(f)$ from a point scatterer has only a phase component $\phi = -2\pi fT$. The relative amplitude spectrum is unity for the case of a point scatterer. Information about the target is only available from the phase information. (The magnitude of the receive signal can, in principle, provide the range to a point scatterer; but it requires knowledge of the values of the parameters of the radar range equation, including the target cross section.)

If the transmitted phase $\phi(t)$ is taken to be zero, the received signal (with ρ = 1 and a(t) constant) is

$$s_r(t) = a \sin 2\pi f(t-T) = a \sin(2\pi ft - 2\pi fT) \tag{6}$$

so that the received phase is $\phi = 2\pi fT$. This is the same as the phase of the received signal spectrum.

When the target is a distributed scatterer, the amplitude and phase of the normalized spectrum of the received signal is more complicated than indicated above and provides information as to the distributed nature of the scatterer. The subject of the present report, however, is the single point-scatterer.

Appendix III - Radiation Pattern of a Two-Element Interferometer

Consider two antennas, each of dimension D, separated by a distance d, as in the interferometer antenna in Fig. 3 of the text. In this appendix, the intensity radiation pattern in one dimension is derived for the interferometer, which is Eq. 14 of the text. It is assumed that the illumination is constant (uniform) across each of the two antennas of dimension D. The angle ξ measured from the normal is expressed here as $u = \sin \xi$. The field intensity pattern in the far field is

$$g(u) = \frac{1}{\lambda}\int_{-(d+D)/2}^{(-d+D)/2} e^{j2\pi(x/\lambda)u}\,dx + \frac{1}{\lambda}\int_{(d-D)/2}^{(d+D)/2} e^{j2\pi(x/\lambda)u}\,dx$$

<u>1st integral</u>

$$I_1 = \left.\frac{e^{j2\pi(x/\lambda)u}}{j2\pi u}\right|_{-(d+D)/2}^{(-d+D)/2} = \frac{e^{j\pi(-d+D)u/\lambda} - e^{-j\pi(d+D)u/\lambda}}{j2\pi u}$$

$$= \frac{e^{-j\pi(d/\lambda)u}\left(e^{j\pi(D/\lambda)u} - e^{-j\pi(D/\lambda)u}\right)}{j2\pi u}$$

$$= \frac{e^{-j\pi(d/\lambda)u}}{\pi u}\sin[\pi(D/\lambda)u]$$

2d integral

$$I_2 = \left. \frac{e^{j2\pi(x/\lambda)u}}{j2\pi u} \right|_{(d-D)/2}^{(d+D)/2} = \frac{e^{j\pi(+d+D)u/\lambda} - e^{-j\pi(-d+D)u/\lambda}}{j2\pi u}$$

$$= \frac{e^{+j\pi(d/\lambda)u}\left(e^{j\pi(D/\lambda)u} - e^{-j\pi(D/\lambda)u}\right)}{j2\pi u}$$

$$= \frac{e^{+j\pi(d/\lambda)u}}{\pi u} \sin[\pi(D/\lambda)u]$$

$$I_1 + I_2 = \sin\pi(D/\lambda)u \; \frac{e^{+j\pi(d/\lambda)u} + e^{-j\pi(d/\lambda)u}}{\pi u}$$

$$= \frac{\sin \pi(D/\lambda)u}{\pi u/\lambda} \; 2 \cos \pi(d/\lambda)u$$

The first factor (sine) is the pattern of an antenna of dimension D. The second factor (cosine), including the factor of 2, represents the interferometer pattern of isotropic radiators, which is $2 \cos\pi(d/\lambda)u$. This is the same as Eq. 14 of the text if the first factor is identified with $g_e(\xi)$, the pattern of the individual antenna. If the difference of the two antennas had been taken, instead of the sum, the sign of I_2 would be minus and the interferometer pattern would be $-2j \sin\pi(d/\lambda)u$. Thus the difference produces a sine spatial pattern, the sum produces a cosine spatial pattern.

In Sec. 3 of the text, the spatial phase was taken as the phase difference, $\phi_s = 2\pi(d/\lambda)u$, between the two antennas of the interferometer rather than as the phase of the sum of the signals. The phase of the sum (or the difference) of the signals from the two antennas of the interferometer depends on both the range and the angle. Therefore, there is coupling between the range and angle measurements based on the partial derivatives of the received phase. This is shown below.

The transmit signal from the interferometer is taken as $\sin 2\pi ft$. The signal received at one antenna is $\sin 2\pi f(t-T)$ and the signal received at the other antenna is $\sin [2\pi f(t-T) + \phi_s]$, where $T = 2R/c$, R = range, c = velocity of propagation, and $\phi_s = 2\pi(x/\lambda)u$. (The two elements of the interferometer are assumed, for convenience, to be isotropic radiators, and the spacing d is taken as the variable x.) The sum of the signals from the two

antennas is

$$\text{sum} = 2 \cos [\pi(x/\lambda)u] \sin [2\pi f(t-T) + \pi(x/\lambda)u]$$

$$= 2 \cos [\pi(x/\lambda)u] \sin [2\pi ft + \phi_a]$$

where $\phi_a = 4\pi(f/c)R + \pi(f/c)xu$

If we take the partial derivative of ϕ_a with respect to the spacing x, we get the angle of arrival ξ, as expected. Thus

$$\partial\phi_a/\partial x = \pi(f/c) \sin \xi$$

However, the partial derivative with respect to frequency f gives

$$\partial\phi_a/\partial f = (4\pi/c)R + \pi(x/c) \sin \xi$$

This provides the sum of the range and the angle, and neither can be extracted separately. The partial derivative with respect to the angle ξ is

$$\partial\phi_a/\partial u = 4\pi(f/c)\ \partial R/\partial u + \pi(f/c)x$$

since the range $\bar{R}$ depends on the angle ξ ($u = \sin\xi$). This has no obvious interpretation. The partial of ϕ_a with respect to time is

$$\partial\phi_a/\partial t = (4\pi/\lambda)\ \partial R/\partial t + \pi(x/\lambda)\ \partial u/\partial t$$

which combines both the range rate $\partial R/\partial t$ and the angle rate $(\partial u/\partial t)$.

Thus the phase term of the sum (or difference) signal from the interferometer does not provide the target information we seek. The information is in the amplitude, or $2 \cos \pi(d\lambda)u$, which can be considered the <u>spatial signal.</u> It is the phase of the spatial signal as a function of spacing x and time t which provides, respectively, the angle and angle-rate. It appears only to be fortuitous that the partial of the temporal phase with respect to x provides the same (angle) information as the partial of the spatial phase with respect to x. The angle rate can only be obtained from the spatial phase.

Appendix IV — Selection of Pulse Intervals for Accurate, Unambiguous Doppler Frequency Measurement

This discussion extends that presented in Sec. 7 for selecting the frequencies for the accurate, unambiguous determination of range. The problem considered in this Appendix is the selection of the successive pulse intervals for the accurate, unambiguous measurement of the doppler frequency (and, hence, the radial velocity) in an MTI radar. The method described here differs from the normal staggered pulse repetition interval waveform often used in MTI radar.

The waveform consists of a succession of pulse intervals $T_1, T_2, \ldots, T_N$. The smallest interval T_1 between pulses (the largest prf, f_1) is chosen so that there are no doppler ambiguities (no blind speeds). The prf is

$$f_1 = 1/T_1 = 2f_{dmax} = 4v_{max}/\lambda \qquad (1)$$

where f_{dmax} = maximum unambiguous doppler frequency that is expected, v_{max} = first blind speed, and λ = wavelength. The factor of 2 that multiplies f_{dmax} in Eq. 1 is due to the Nyquist sampling theorem that requires at least two samples per period of the highest frequency for unambiguous frequency measure ment. (If the concern is to avoid blind speeds rather than unambiguous frequency measurement, the factor of 2 would be omitted.) The largest interval between pulses T_N (the smallest prf, f_N) is determined by the need for unambiguous range R_{un}. Thus

$$f_N = 1/T_N = c/2R_{un} \qquad (2)$$

The ratios of the other prfs (f_2/f_1, f_3/f_2, etc.) can be taken to be 10 to 1.

As an example, assume the maximum radial velocity (first blind speed) is v_{max} = 2000 kn and that the wavelength λ = 23 cm (L band). From Eq. 1, the largest prf is 17.4 kHz (T_1 = 57.5 μs), corresponding to an unambiguous range of 8.63 km (4.66 nmi). If the unambiguous range is assumed to be 240 nmi, the lowest prf as given by Eq. 2 is 337 Hz. One additional prf is needed. This is selected (somewhat arbitrary) as the geometric mean of the other two values already selected, which is 2.42 kHz (instead of the 10 to 1 ratio since the ratio of f_N/f_1 in this case is less than 10). The unambiguous range for this prf is 62 km, or 33.5 nmi. Thus this waveform contains four pulses with three spacings: T_1 = 5.75 μs, T_2 = 413 μs, and T_3 = 3 ms. This waveform has both range and doppler ambiguities, but it should be possible to resolve them. It would be of interest to examine the classical ambiguity diagram produced by this waveform.

A waveform commonly used for a long-range L-Band radar consists of a train of five pulses with four different intervals. Unlike the waveform described above, each of the four intervals provides an unambiguous range measurement. The intervals might, for example, be in the ratio 25:30:27:31, with the shortest interval corresponding to the maximum unambiguous range. The method for selecting pulse intervals described here should have less loss than the conventional staggered interval waveform since it has a large ratio of max to min spacing. Range ambiguities result, but it should be possible to resolve them.

Appendix V - Phase Derivatives in Other Measurement Methods

FM-CW Radar. A CW radar does not measure range unless its waveform is modulated in frequency, phase, or amplitude so as to increase its spectral bandwidth. Frequency modulation is the usual method, as is done in the classical radio altimeter.

The phase of the returned signal is given by Eq 3 of the main body of the report as

$$\phi = 2\pi f T \tag{1}$$

where f = frequency, T = 2R/c = round-trip time delay to the target at range R, and c = velocity of propagation. The frequency f is assumed to be a function of time. The time delay T can also be a function of time ($\partial T/\partial t$ is a measure of velocity). Normally, the derivative of phase with respect to time ($\partial\phi/\partial t$) gives the doppler frequency shift, and the radial component of target velocity. When frequency is not a constant, $\partial\phi/\partial t$ gives the range. (It will be recalled that FM-CW radars extract a frequency from which range is determined.)

Differentiating Eq. 1 with respect to time, we get

$$\partial\phi/\partial t = 2\pi f T' + 2\pi f' T \tag{2}$$

If the first term is small compared to the second term, this equation gives the time delay T (or range) and is

$$\frac{\partial\phi}{\partial t} = 2\pi f' T \tag{3}$$

To see that the first term can be neglected in a practical case, take f = 3000 MHz, T = 0.001 s (R = 80 nmi), Δf = 100 MHz, ΔT = 0.01 sec, ($f' = \Delta f/\Delta T$ = 10 GHz/s), and T' = 1 μs/s (corresponds to a velocity of 150 m/s). Then fT' = 3000 is much smaller than $f'T = 10^7$.

Differential Doppler. In this technique, the doppler frequency is measured at two locations separated a distance x. The measurement provides angle rate. The doppler shift is given by $\partial\phi/\partial t$, or

$$\partial\phi/\partial t = (4\pi/\lambda)v_r \tag{4}$$

where v_r = radial velocity = $\partial R/\partial t$. The differential doppler is

$$\frac{\partial}{\partial x}\frac{\partial\phi}{\partial t} = \frac{\partial}{\partial t}\frac{\partial\phi}{\partial x} \tag{5}$$

If interchanging the order of differentiation is permitted, then Eq 5 shows that differential doppler $\partial^2\phi/\partial x\partial t$ (left hand side of the equation) is the same as the rate of change of angle $\partial\phi/\partial x$ with respect to time.

Time Difference of Arrival (TDOA). The angle of a target can be found by measuring the difference in the time of arrival of signals received at two separate locations. Its advantage is that non-directive antennas can be used. It has application in ESM, but has not been of interest in radar because it does not use directive antennas (with their large receiving aperture). The narrow beamwidths and large apertures associated with directive antennas is important in radar applications. Time and phase are related, so that $\Delta T/\Delta x$ is equivalent to $\Delta\phi/\Delta x$, or angle. The advantage of measuring ΔT instead of $\Delta\phi$ is that wideband signals can be used which avoid the problems of ambiguities and the contamination from multiple signals (targets) that are characteristic of phase measurements.

Appendix VI - Accuracy of Phase-Derivative Measurements

Introduction

The main body of this report discussed the use of phase derivatives for obtaining the target location (range and angle) and change of location (range rate and angle rate). The phase derivatives are not the usual method for obtaining radar measurements. Therefore, this appendix addresses the theoretical accuracy with which phase-derivative measurements can be made and compares them with conventional radar measurements. The emphasis is on the angle-rate measurement since it has not been a usual type of radar measurement. Angle rate can be obtained from the spatial doppler frequency shift or, more conventionally, from two angle-measurements separated by an interval of time.

The accuracy, or rms error, with which radar measurements can be made when the limitation is white, gaussian noise, has been described in a number of references.[1-3] In general, the theoretical accuracy, δm, potentially available from a radar measurement can be expressed in the following general form

$$\delta m = \frac{k}{m\,(2E/N_0)^{1/2}} \tag{1}$$

where k = constant (depends on the shape of the waveform, and is generally of the order of unity), E = received signal energy, N_0 = noise power per unit bandwidth (units of energy), and m is related to the parameter to be measured. The parameter m is defined by the expression

$$m^2 = \frac{(2\pi)^2 \int x^2\, \Phi^2(x)dx}{\int \Phi^2(x)dx} \tag{2}$$

where $\Phi(x)$ is a spectrum or waveform related to the type of measurement to be made. (The parameter m is a form of second moment of the distribution $\Phi(x)$.)

Review of Range-Rate Measurement Accuracy.

The measurement of the more familiar range rate, or radial velocity, will be discussed first so as to provide an introduction to the less well-known angle rate measurement. The measurement of range rate can be made either (1) as the rate of change of range with time, (2) from the doppler frequency shift, or (3) from the rate of change of echo signal phase with time. It will be shown that the measurement of range rate based on either (2) or (3) can be much more accurate than the measurement of range as a function of time.

Method No. 1 - $\Delta R/\Delta T$. In this method, the range rate is found from two measurements of range separated by a time interval T. The theoretical rms error in measuring time delay is[1]

$$\delta T_R = \frac{1}{\beta\sqrt{2E/N_o}} \tag{3}$$

where β is the effective bandwidth defined by Eq. 2 with $m = \beta$, $x = f$, and $\Phi(x) = S(f)$. If the bandwidth B determines the pulse rise time (so that the rise time $\approx 1/B$), the rms error from a single measurement is approximately

$$\delta T_R = \frac{1}{B(4E_1/N_o)^{1/2}} \tag{4}$$

where E_1 is the energy of the signal in a single range measurement. Since range $R = cT_R/2$, where c = velocity of propagation and T_R = time delay, the rms error in measuring a distance R is

$$\delta R = \frac{c}{(2B)\ (4E_1/N_o)^{1/2}} \tag{5}$$

To measure range rate, two range measurements separated by a time $\Delta t = T$ must be made. The rms error in measuring ΔR, the difference in the two range measurements, will be $\sqrt{2}$ times δR. It is assumed there is no error in knowledge of the time separation T. The total energy in the two measurements is $2E_1 = E$. Therefore the rms error in measuring $\Delta R/\Delta t$, or radial velocity, is

$$\delta v_r = \frac{c}{(2)^{1/2}\ BT(2E/N_o)^{1/2}} \tag{6}$$

Method No. 2 - doppler frequency. In this method, the radial velocity is found from the doppler frequency shift. The theoretical accuracy (rms error) in the measurement of frequency is given by

$$\delta f = \frac{1}{\alpha(2E/N_o)^{1/2}} \tag{7}$$

where α = effective time duration (defined by Eq 2 with $x = t$ and $\Phi(x) = s(t)$. For a rectangular pluse of duration T, the effective time duration $\alpha = \pi T/\sqrt{3}$. The radial velocity v_r and the doppler frequency shift f_d are related by

$$v_r = \lambda f_d/2 \tag{8}$$

where λ = radar wavelength. The rms error in measuring the radial velocity

is therefore

$$\delta v_r = (\lambda/2)\delta f_d = \frac{\sqrt{3}\lambda}{2\pi T(2E/N_0)^{1/2}} \tag{9}$$

$$= \frac{\sqrt{3}c}{2\pi f_0 T(2E/N_0)^{1/2}}$$

where c = velocity of propagation and f_0 = radar frequency.

The ratio of Eqs. 6 and 9 represents the rms error in measuring radial velocity by means of $\Delta R/\Delta t$ relative to its measurement from the doppler frequency. This ratio is

$$\frac{\text{rms error from } \Delta R/\Delta t}{\text{rms error from doppler}} = \frac{\text{Eq 6}}{\text{Eq 9}} = \frac{\pi}{\sqrt{1.5}} \; \frac{fo}{B} \tag{10}$$

Since $f_0/B >> 1$, the rms error in measuring radial velocity from the doppler frequency shift can be several orders of magnitude less (that is, better accuracy) than the measurement based on the rate of change of range with time. The improved accuracy of the radial velocity based on doppler frequency measurement with pulse waveforms; however, might be accompanied in practice by amgituities in range or radial velocity, or both.

<u>Method No. 3 - $\Delta\phi/\Delta t$.</u> This is the method described in Sec. 3 of the main body of the report. From Eq. 4 in the main body, the radial velocity is

$$v_r = (\lambda/4\pi)(\delta\phi/\delta t) = (\lambda/4\pi)(\Delta\phi/T) \tag{11}$$

The phase derivative is found by measuring two phases separated by the time T. It is assumed that the rate of change of phase with time is linear, that the time separation T is known without error, and that the rms error in measuring phase is given by[3]

$$\text{rms phase error} = (2E/N_0)^{-1/2} \tag{12}$$

Since two phases have to be measured to find $\Delta\phi$, the rms error in $\Delta\phi$ is 2 times that given by the single measurement represented by Eq 12. (A factor of $\sqrt{2}$ is included because two measurements are made. Another $\sqrt{2}$ factor accounts for the total energy, a similar argument as given for Eq. 6.) Therefore, the rms error in measuring the radial velocity is

$$\delta v_r = \frac{\lambda}{2\pi T} \frac{1}{\sqrt{2E/N_0}} \tag{13}$$

This is the same as the rms error of Eq. 9 for the radial velocity found from the doppler frequency, except that Eq. 13 is smaller (better accuracy) by a factor of $\sqrt{3}$. Generally, constant factors of this magnitude have not been of concern in the past,[3] so that we will say that the accuracy of the measurement of radial velocity by the rate of change of phase with time is essentially the same as that found by the doppler frequency shift (which seems intuitively correct.) It might be noted that the equivalent of matched-filter measurement of phase has not been described in the literature, as it has for the matched-filter detection of time delay and doppler frequency shift.

Angle-Rate Measurement Accuracy

As with the measurement of range rate, the angle-rate measurement can be made by (1) the rate of change of angle with time, (2) the spatial doppler frequency shift, and (3) the rate of change of spatial phase with time. As with range rate, methods (2) and (3) are similar.

Method No. 1 - $\Delta\theta/\Delta t$. This is the measurement of two angles separated by a time T. The accuracy in measuring angle is given by[1]

$$\delta\theta = \frac{1}{\gamma\sqrt{2E/N_0}} \tag{14}$$

where γ = effective aperture size (in wavelengths), which is given by Eq. 2 with x = aperture parameter in wavelengths, and $\phi(x) = A(x)$, the aperture illumination. With a conventional antenna and a uniform aperture illumination, $\gamma = \pi D/\sqrt{3}\,\lambda$, where D = physical aperture size. The angle rate in Method No. 1 is found by making two independent measurements of angle separated by a time T. The time T is assumed to be known precisely. The rms error in measuring the angle rate $\Omega_t = \Delta\theta/T$ is

$$\delta\Omega_t = \frac{2\sqrt{3}\,\lambda}{\pi DT\sqrt{2E/N_0}} \tag{15}$$

The factor of 2 in the numerator results because two measurements of angle are needed to find $\Delta\theta$, and the total energy is twice that of a single measurement.

Method No. 2 - spatial doppler frequency. The spatial doppler frequency shift is given by Eq. 20 of the text for a scanning interferometer, which is

$$f_s = (d/\lambda)(v_c/R) = (d/\lambda)\Omega_t \tag{16}$$

where d = spacing between interferometer antennas, λ = wavelength, v_c = cross velocity, R = range, and $v_c/R = \Omega_t$ is the angle rate of the target. The accuracy with which frequency can be measured was given by Eq. 7 of this appendix.

Assuming that the spatial signal of frequency f_s has a duration T, the value of α, the effective time duration, is $\alpha = \pi T/\sqrt{3}$. The duration T is equal to $\lambda/D\Omega$, where D = antenna dimension, and Ω = angular scan rate of the interferometer pattern (radians/second). From Eqs. 7 and 16, the rms error in measuring the angle rate, based on the spatial doppler frequency, is

$$\delta\Omega_t = \frac{\sqrt{3}}{\pi} \frac{\lambda}{dT\sqrt{2E/N_0}} = \frac{\sqrt{3}}{\pi} \frac{D\Omega}{d\sqrt{2E/N_0}} \tag{17}$$

The ratio of the rms error in measuring angle rate based on $\Delta\theta/\Delta t$ (Eq. 15) to the error in angle rate based on spatial doppler (Eq. 17) is

$$\frac{\text{rms error from } \Delta R/\Delta t}{\text{rms error from doppler}} = \frac{\text{Eq 15}}{\text{Eq 17}} = \frac{d}{D} \tag{18}$$

Since the spacing d between antennas is larger than the antenna dimension D, the accuracy of the spatial doppler is better than that from $\Delta\theta/\Delta t$. The larger the value of d/D, the better is the accuracy of the spatial doppler method. (The ratio of Eq. 18 assumes the same total value of E in both cases. If two antennas are assumed in the interferometer in making the spatial doppler, but only one for an angle measurement, the energy E in Eq. 15 is one-half that of Eq. 17. Hence, the ratio of Eq. 18 would be $\sqrt{2}d/D$ if the antenna diameters are the same.)

The extraction of the angle rate from the spatial doppler frequency shift is seen to be more accurate than the conventional measurement of angle rate based on $\Delta\theta/\Delta t$. The difference, however, in the two methods is not as large as the range rate methods since d/D is usually not as great at f_0/B in Eq. 10.

Method No. 3 - $\Delta\theta_s/\Delta t$. Eq. 10 of the main body of the report gives the angular rate as

$$\Omega_t = \frac{v_c}{R} = \frac{\lambda}{4\pi x} \frac{\Delta\phi_s}{T} \tag{19}$$

where the angle rate is obtained as the measurement of phase at two times separated by the interval T. The rms error in measuring Ω_t is

$$\delta\Omega_t = \frac{\lambda}{4\pi d} \frac{2\,\delta\phi}{T} \tag{20}$$

where x is taken to be equal to the antenna separation d of the scanning interferometer. The factor 2 is included since two phase measurements are made and the total energy is twice the energy of a single measurement. The value of $\delta\phi$ is (as before) $(2E/N_0)^{-1/2}$. Then Eq. 20 becomes

$$\delta\Omega_t = \frac{\lambda}{2\pi dT\sqrt{2E/N_0}} \tag{21}$$

This is similar to that given by Eq 17 for the error obtained from the doppler frequency shift, but it is smaller by a factor $\sqrt{12}$.

Range-Measurement Accuracy.

Equation 4 in this appendix gives the rms error in measuring time delay T_R as

$$\delta T_R = \frac{1}{\sqrt{2}\ B\ \sqrt{2E/N_0}} \qquad (22)$$

where B = bandwidth. From Section 3 of the main body, the time delay T_R obtained from the rate of change of phase with frequency is

$$T_R = \frac{1}{2\pi}\ \frac{\partial\phi}{\partial f} \qquad (23)$$

If $\partial\phi/\partial f \approx (\phi_2-\phi_1)/(f_2-f_1)$ and f_2-f_1 is made equal to the bandwidth B, then

$$\delta T_R = \frac{1}{\pi B\ \sqrt{2E/N_0}} \qquad (24)$$

assuming, as before, a factor of 2 degradation in accuracy because of the two measurements, Eq 24 is of the same general form as the time delay accuracy of Eq 22, except that it is less by a factor of $\pi/\sqrt{2}$. The slightly slightly better accuracy of the $\Delta\phi/\Delta f$ measurement (Eq. 24 as compared with Eq. 22) comes about because the effective bandwidth β for time delay is largest when the spectral energy is concentrated at the edges of the band, as it is in the two-frequency measurement of range.

Angle-Measurement Accuracy

The rms error in measuring angle with a uniform illumination over the aperture D is given by Eq 14 with $\gamma = \pi D/\sqrt{3}\lambda$, or

$$\delta\theta = \frac{\sqrt{3}\ \lambda}{\pi D\sqrt{2E/N_0}} \qquad (25)$$

The angle measurement, from Eq 8 of the main body, can be expressed as

$$\sin\ \xi \approx \xi = \frac{\lambda}{4\pi}\ \frac{\Delta\phi_s}{\Delta x} \qquad (26)$$

Letting $\xi = \theta$, $\Delta x = D$, and accounting for the factor of 2 degradation in accuracy, we then have

$$\delta\theta = \frac{\lambda}{2\pi D\sqrt{2E/N_0}} \qquad (27)$$

which is similar to Eq 25, except for a factor of $\sqrt{12}$.

References, Appendix VI

1. Skolnik, M.I.: Introduction to Radar Systems, McGraw-Hill Book Co., New York, N.Y. 1980, Sec. 11.3.

2. Barton, D. K. and H. R. Ward: Handbook of Radar Measurement, Prentice-Hall, Inc., Englewood Cliffs, N.J. 1969.

3. Swerling, P.: "Radar Measurement Accuracy," Chap. 4 of Radar Handbook, Mc-Graw-Hill Book Co., New York, N.Y., 1970.